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PHOTOGRAPHY IN THE COLORS OF NATURE.

BY F. E. IVES.

[*A Lecture delivered before the Franklin Institute, December 19, 1890.*]

THE lecturer was introduced by Prof. EDWIN J. HOUSTON, of the institute, and spoke as follows:

MEMBERS OF THE INSTITUTE AND LADIES AND GENTLEMEN:

I have already made two communications to the Franklin Institute on the subject of photography in colors. My object in lecturing upon it at this time is to go further into its history, to give such accurate knowledge of the subject as will make the force of my criticisms more readily apparent than heretofore, and to better demonstrate my own method. The substance of my lecture might have been more appropriately presented in the form of a paper to be read at a stated meeting of the institute; but a satisfactory presentation of a subject so complex and difficult could not be

made in the brief space of time allowed for the reading of such papers, and I offer this excuse for presenting it as a lecture.

Heliochromy—meaning sun-coloring—has been settled upon as a name for processes of photography in natural colors, or in the colors of nature. There are two kinds of heliochromic processes. In one, the light itself produces the colors, by direct action upon the sensitive plate. In the other, light does not produce colors, but is made to regulate their distribution and combination. Some of the colors of the spectrum were imperfectly reproduced by a process of the first kind nearly thirty years before the discovery of the daguerrotype process. Seebeck, of Jena, in 1810, found that chloride of silver, after preliminary exposure to white light, is colored a brick-red by prolonged exposure to the red light of the spectrum, and a metallic blue by the blue light. After the discovery of the daguerrotype process, several experimentalists tried so to modify the preparation of the chloride of silver plates as to make them capable of reproducing all the colors of nature. In a photographic text-book, published so long ago as in 1853, I find the following statement: "Even the long-debated question of the reproduction of the natural colors by the agency of light seems on the point of solution. * * * M. Neipce de St. Victor, from whose well-known character as an experimental philosopher much might be expected, has forwarded to London, as we understand, specimens of proofs in which every color is reproduced with a vigor and richness truly wonderful." Similar announcements have been made since that time, but the best results ever actually shown were nothing more than interesting curiosities. Dr. H. W. Vogel,* who recently had an opportunity to compare some of the latest and most talked-about of these "photographs in natural colors" with the original colored pictures from which they were printed (by contact), says:

"The original is one of those transparent window pictures in bright colors, brought into market by Grimme and

* *Anthony's Bulletin*, 1890, p. 325.

Hembel, in Leipsic, as a substitute for glass painting. It represents a Cupid with yellowish-brown hair and wings, and a small blue scarf around the waist, whose ends wave in the wind. He carries an arrow piercing two hearts of ruby color; between the knees he holds a quiver with yellow ornamented opening, and in his left hand the upper part of a large brown cross-bow, with blue and yellow mountings, the lower part of which rests, with the figure, upon an idealistic thistle blossom of red leaves. The stem is of the same color, and the plant shows fresh green leaves. The picture has a pale-blue background, and red, green and yellow ornamentation around the border in very pronounced colors. This border ornamentation affords an excellent means of comparison with the print. The latter, in opposition to the bright original, shows a greenish-gray, partly dark, ground. At first look, one recognizes readily that of all the colors only the red of the original has been distinctly reproduced. But it is not true to nature; it has a copper-red color, and differs decidedly from the vermilion and carmine red of the original. Besides this copper-red, only the blue of the scarf and the mountings of the cross-bow and quiver comes out as a very pale light-blue, with no natural resemblance. The black lines of the border decoration appear alongside of this as a violet-black. These are the tones which to some extent have a similarity of color, but with the other colors it is not so favorable. The yellow squares and green trapezoids of the border decoration appear neither yellow nor green, but have a grayish-red tone. The blue fields are not blue, but greenish-gray, like the ground. It is most singular that several parts are reproduced in red, which actually are not red, but brown yellow, as, for instance, the hair, the wings, the cross-bow, the thistle, etc. The green leaves in the print show no fresh color, and the red leaves of the blossom and the body of Cupid show only a pale flesh color. * * * The resemblance of the new photographic pictures to natural colors is, therefore, not very favorable. Only two colors can be recognized distinctly in the copy, of which the red is the best; in a less degree the blue, which is weaker as

far as the picture is concerned. The blue in the ornamentation around the border, and all other colors, either have not been reproduced at all, or are entirely unlike the original. * * * If I compare the sample before me with the pictures I have seen in 1867 of Niepce de St. Victor, Becquerel and Dr. Zenker, I must confess that those much older productions were richer in color, although the tones deviated likewise considerably from the natural ones."

According to Capt. Abney, the red end of the spectrum produces red by promoting oxidation; the blue end, blue, by its reducing action.* Prof. Mendola† says: "It may at first sight appear improbable that the coincidence between the colors of the spectrum and the colors of the impressed film is a mere accident; but, although this is difficult to believe, I venture to think that it is an accidental coincidence and nothing more. * * * In the best specimens of these photochromatic spectra that I have seen, the colors were certainly nothing more than approximations to the pure spectrum colors; and even in these spectra, some of the colored effect was due to the unaltered ground-color of the film in regions where some particular color had produced no action at all."

The process by which such imperfect results have been obtained is too slow to be applied successfully to camera photography, and the results are not permanent.

In view of all these facts, it would appear that there is no scientific basis for a belief that any material improvement can ever be made in this process, and that all so-called progress along this line is a delusion. It is true that some distinguished photographic writers continue to regard every new modification of this old process and every new result of experiment with it, as another step towards the photographic reproduction of the natural colors; but I have no doubt that if the same writers had lived 200 years ago, they would have regarded the production of new yellow-colored metal alloys as steps towards the transmutation of the baser metals into gold.

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* *Anthony's Bulletin*, 1890, p. 307.

† *Chemistry of Photography*, p. 324.

In my opinion, the first step towards the solution of this problem was taken by Henry Collen, queen Victoria's painting-master, who, in 1865, invented a plan of composite heliochromy. His plan was based upon a false conception of the nature of color, and means for carrying it out were then unknown; but it was a bright idea, and contained the germ of a successful process. Collen's original communication of his idea appeared in the *British Journal of Photography*, October 27, 1865, and reads as follows:

"It occurred to me this morning that if substances were discovered sensitive only to the primary colors—that is, one substance to each color, it would be possible to obtain photographs with the tints as in nature by some such means as the following:

"Obtain a negative sensitive to the blue rays only; obtain a second negative sensitive to the red rays only, and a third sensitive to the yellow rays only.

"There will thus have been three plates obtained for printing in colors, and each plate having extracted all its own peculiar color from every part of the subject in which it has been combined with the other two colors, and being in a certain degree analogous to the tones used in chromolithography. Now, it is evident that if a surface be prepared for a positive picture, sensitive to yellow rays only, and that the two negatives, sensitive only to blue and red, be superimposed either on the other, and be laid on this surface, the action of light will be to give all the yellow existing in the subject, and if this process be repeated on other surfaces sensitive only to red or blue, respectively, there will have been produced three pictures of a colored object, each of which contains a primitive color reflected from that object.

"Now, supposing the first great object achieved, viz: the discovery of substances or preparations, each having sensitiveness to each of the primary colors only, it will not be difficult to imagine that the negatives being received on the surface of a material quite transparent and extremely thin, and that being so obtained are used as above—*i. e.*, each pair of superimposed negatives to obtain the color of the third—that three positives will be obtained, each represent-

ing a considerable portion of the form of the object, but only one primary of the decomposed color of it. Now, if these three positives be received on the same kind of material as that used for the negatives, and be then laid the one on the other, with true coincidence as to the form, and all laid upon a white surface, it will not be difficult to imagine that the effect would be, not only the representation of the form of the object, but that of its color also, in all its compounds.

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"Although the idea I have endeavored to express in words may be utterly worthless, I am unwilling to let it slip away without notice, as it may on the other hand contain a germ which may grow and bear fruit in due season."

The language of some parts of this communication is ambiguous; but, taken all together, with due allowance for the writer's unfamiliarity with photographic technology, it clearly amounts to a suggestion to make three photographic negatives of an object—one by the action of red light, one by yellow, one by blue; to print from each pair of these negatives (superposed as one) a transparent positive having the color represented by the third negative, and to superpose on a white surface the three prints thus obtained.

It was not possible to carry out Collen's suggestion at that time, because there was no known process by which plates could be prepared which were sensitive to single colors only, and no photographic plates were sensitive enough to red and yellow to admit of the production of such negatives by exposure through selective color screens. Had it been possible to carry it out, the results must have been very imperfect, not only because the entire procedure is based upon a false and misleading theory of color, but also because superposing two negatives to act as one would double the intensity of such parts as represented white, gray or pale-colored objects, with the result that if the color prints were made to show all the details of the negatives, the finished heliochromes would show all bright colors as if mixed with equal parts of black pigment.

On November 23, 1868, Ducos Duhauron, of Paris, applied for a patent* for a process which differed from Collen's only in the manner of carrying out the same idea. Like Collen, he assumed that the spectrum is made up of three primary color rays and mixtures thereof. He said: "My procedure rests on the principle that the simple colors are limited to three—the red, the yellow and the blue—the combination of which in divers proportions produces the infinite variety of shades in nature." Like Collen, he expected to solve the problem by superposing red, yellow and blue prints taken from negatives made by yellow and blue, red and blue, and yellow and red light. But, instead of using plates sensitive to single colors only, he proposed to use plates sensitive to all colors, and to prevent the action of color rays not wanted by filtering them out with color screens placed in front of the photographic objective or sensitive plate; and, instead of superposing two negatives to act as one, from which to make the color prints, he proposed to make two colors (two-thirds of the spectrum rays) act to produce each negative, which amounts to the same thing, and would not obviate the defect I have mentioned as resulting from the doubling of intensity on uncolored objects. He proposed to make one negative through an "orange" screen, calculated to absorb the blue light and transmit the red and yellow; one through a "violet" screen, calculated to absorb the yellow light and transmit the blue and red; one through a "green" screen, calculated to absorb the red light and transmit the yellow and blue.

It was no more possible to carry out this idea in Duhauron's way in 1868 than to carry it out in Collen's way in 1865. It is true, Duhauron tried to carry it out, and showed specimens of work, but the red and yellow rays did not act on his sensitive plates,† and he admitted, in a communication to the French Photographic Society,‡ that "the

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* Class xvii, Sec. 3, Serial No. 83,061.

† Yellow pigments were photographed by the green rays which they reflected.

‡ *Photo. News*, 1869, p. 319.

production of good results will * * * involve the manufacture of compounds which have not yet been created."

Soon after Duhauron showed his first specimens, Charles Cros, of Paris, published another modification of Collen's plan.* Like Collen, Cros proposed to make one negative by the action of red light, one by yellow, and one by blue; but by exposing the sensitive plates through red, yellow and blue screens instead of employing plates sensitive to single colors only. Instead of superposing each pair of these negatives to make each color print, he proposed to make a green print from the negative made by red light, a violet print from the negative made by yellow light, and an orange print from the negative made by blue light. He also suggested that ordinary positive prints made from these negatives might be illuminated each by the kind of light which it represented, and the three combined by the aid of suitable optical devices so as to form a single picture, showing all the colors. Cros' plan, although it could not succeed because based upon the same false and misleading theory as that accepted by Collen and Duhauron, nevertheless possessed one important advantage over the preceding methods: it was free from the defect of doubling intensity on those parts of the negatives representing pale or uncolored objects. But this advantage would be lost again, in the production of green, violet and orange-colored prints, which will combine to reproduce yellows and blues only with a degree of degradation comparable to that produced by Duhauron's method.

On Dec. 3, 1869, M. Poirée, of Paris, in a communication to the Photographic Society of France,† expressed doubts concerning the correctness of Duhauron's and Cros' theories, and suggested that better results might be had by making a greater number of negatives—a separate negative for each spectrum region. He said: "The process which seems likely to succeed best is that in which the colors are analyzed by isolating successively each ray, or at least the rays of the

* Described in *Photographic News*, Oct. 8, 1869, p. 483.

† *British Journal of Photography*, 1870, p. 26.

same shade. * * * This analysis is difficult to make with colored glasses; it might be done, as by Newton, by monochromatic lighting and successive exposures to simple rays of the same shade. * * * The synthesis is made by means of black positive images and rays of the same nature as those which produced the corresponding negatives. * * * It will then only be necessary to place one above another the colored images so obtained, so as to form one virtually and really. It will be identical with the model, because it will be formed by the same rays, in the same relation of intensity." This plan also could not then be carried out because no photographic sensitive plates were sufficiently sensitive to yellow, orange and red spectrum rays.

In 1873, Dr. H. W. Vogel discovered that bromide of silver can be made sensitive to the less refrangible spectrum rays by treatment with certain dyes, and the subsequent discovery of other and better color sensitizers supplied the means for carrying out either Collen's or Poirée's idea.

Duhauron, one of the first to avail himself of these discoveries, made some practical progress, and, in 1876, abandoned Brewster's color theory and patented a modified process,* based upon the observation that, while there appeared to be *seven* "principal" spectrum colors, three coloring substances would "serve to express them." The coloring substances he named for this purpose are blue, carmine and yellow, and he decided that, in order to make such a process reproduce the colors of nature, the negatives should be made by the action of orange, green and violet spectrum rays, which are complementary to the coloring substances. Some persons have thought that he had the idea of making negatives to represent primary color sensations; but this supposition is negatived, not only by the absence of any declaration to that effect, but also by the fact that orange does not represent a primary color sensation, either in fact or according to any theory recorded in the text-books, and the violet rays are not the ones which

* British patent, July 22, 1876, No. 2,973.

most powerfully excite the blue (violet) sensation. The plan was also utterly indefinite as regards the relative effect of intermediate spectrum rays, and Dehauron himself, owing to the fact that he never tried the method upon the spectrum, had no accurate knowledge of its capabilities. In his latest and "perfected" process (1878),* he employed no plate sensitive to either red or orange light; one negative was made chiefly by yellow light, another by green, and the third chiefly by violet and invisible ultra-violet rays.

Albert, of Munich, also took advantage of the discovery of color sensitizers to try to carry out Collen's principle according to Duhauron's original plan. He was the first to make the color prints by the collotype process, which led to the use of the term "chromo-collotype."

In 1879, Cros† abandoned the idea that red, yellow and blue are primary spectrum colors, but still held that there are three primary colors and mixtures thereof, and that these primary colors are orange, green and violet. Like Duhauron, he decided to make negatives by light of these colors and prints in blue, red and yellow.

In 188–, Dr. F. Stolze, of Berlin, made a series of investigations, and tried to solve the problem by devising a procedure more in accordance with Young's theory of color.‡ He said: "Although the colors correspond with certain external processes in nature, there is also no doubt that color as such is nothing objective, but a subjective sensation, based upon the peculiar irritation of the visual nerves by those external proceedings. We can, therefore, only hope to produce a picture in natural colors when we are enabled to reproduce upon the same the proceedings which furnish to us the color impression." "The general idea of all colors being based upon the three principal colors, red, yellow and blue, is an erroneous one." "Theodore Young * * * assumes that there are three kinds of nerve fibres sensible

* "Traité Pratique de Photographie des Couleurs," Paris, 1878, *Photo. News*, 1878, p. 115.

† *Bulletin of the French Photographic Society*, 1879, p. 23.

‡ *Anthony's Photographic Bulletin*, 1888, pp. 516, 555, 588, 647, 678.

to red, green and violet. Objective homogeneous light excites all three; but with red the first is excited strongly, the second and third weakly; with blue, the second and third moderately strong, the first weakly; with violet finally, the third strongly and the first and second weakly. If all three kinds of nerve fibres are equally strongly excited, the impression of white light will take place."

This theory, in accordance with which Dr. Stolze tried to devise a theoretical solution of the problem, is only partly correct, measurements by Clerk Maxwell and others having shown that the "red" sensation is neither affected by blue-green, blue or violet rays, nor the "blue" (violet) sensation by red, orange or yellow rays, nor the "green" sensation by red or violet rays. Neither is it the red rays that chiefly excite the red sensation, nor the violet rays that chiefly excite the blue (violet) sensation.

As a result of elaborate calculations, which, it must be said, could just as well have been made without any reference to Young's theory of color, Dr. Stolze came to the conclusion that if three suitable selective color screens were used in connection with color-sensitive plates, three negatives of the spectrum might be obtained, from which prints in cyan blue, carmine and yellow, if superposed, would reproduce the color effect of the spectrum. He did not show how to make selective color screens calculated to secure the right kind of negatives to carry out this idea, nor state what should be the form of the intensity curves in such negatives of the spectrum. He merely gave a table, showing on what parts of the spectrum each negative should fix color, and said: "If successful * * * in selecting the color screens in such a manner that they will let the colors pass through which are called for in this table, one will indeed be able to reproduce a pure spectrum in this way." By further calculations, he was able to show that this plan, even if successfully carried out, would not insure the correct reproduction of mixed colors. He said: "All pure saturated spectrum colors will also be obtained quite satisfactorily in the reproduction, but the mixed ones only partly." "Oftentimes they have to become more or less

impure." "But the clearest lights and a number of mixed colors appear very unsatisfactory." He added: "The intelligent support of the artist can lend improvement," and recommended also the production of a fourth (ordinary) negative, to be used in combination with the others, to modify the effect, especially in the high lights.

This plan cannot be said to definitely represent the application of Young's theory of color, but it may be practically better than anything that that theory would indicate, if we leave out of account the suggestion of a fourth negative.

In 1885, Dr. Vogel, published a plan which is a modification of Poirée's.* Like Poirée, he proposed to make a separate negative for each spectrum region; but, instead of using plates sensitive to all colors and exposing through selective color screens, or illuminating the subject by monochromatic lights, Vogel proposed to sensitize plates specially for each spectrum region, which would amount to the same thing; and instead of projecting the pictures with colored lights, he proposed to make as many pigment prints as negatives, each in a color complementary to the light which acted to produce the respective negative, and to surperpose them as in the Collen method.

There are no known dyes with which this plan could be carried out, and even if there were, it is, I believe, too complicated to be practicable.

In February, 1888,† I demonstrated a procedure based upon the assumption that, although there are more than three or five or seven primary spectrum colors, all of them—and in fact all the colors of nature—can be counterfeited to the eye by three type colors and mixtures thereof. This was not a new observation, and my plan did not differ very materially from that of Dr. Stolze, minus the complication of a fourth negative, except that it was more definite; and instead of merely publishing it as a suggestion, I found means to carry it out, and made a practical demonstration of it. I proved the process by photographing the spectrum

* *Annalen der Physik* (*N. S.*,) xxvii, p. 130; *Photo. News*, 1887, p. 568.

† *Journal of the Franklin Institute*, **125**, 345.

itself, employing compound color screens carefully adjusted to secure definite intensity curves in the spectrum negatives, so that they would make color prints which counterfeited the color effect of the spectrum when superposed. The adjustment of plates and screens to secure *spectrum negatives having definite intensity curves*, which, I believe, had never before been done, made all the difference between an indefinite and uncertain method and one definite and precise.

Promising results were obtained by this process, but I soon came to the conclusion, already reached by Dr. Stolze, that a process might reproduce the color effect of the spectrum, and yet not be capable of reproducing perfectly the compound colors. The solution of the problem was incomplete until I discovered a new principle, according to which such a procedure can be made to reproduce not only the spectrum, but also all the hues of nature.

This new principle, first stated by me in a communication to this institute on November 21, 1888,* is that of making sets of negatives by the action of light rays in proportion as they excite primary color sensations, and images or prints from such negatives with colors that represent primary color sensations.

In order to understand this principle, I must explain that although the spectrum is not made up of three kinds of color rays and mixtures thereof, the eye is only capable of three primary color sensations—a distinction of the utmost importance, for the reason that the spectrum rays, which most powerfully excite a primary color sensation, are not the ones which represent the character of that sensation. The primary sensations are red, green and blue (violet); but it is not the red, green and violet spectrum rays that most powerfully excite these sensations. According to Clerk Maxwell, the orange spectrum rays excite the red sensation more strongly than the brightest red rays, but also excite the green sensation; the greenish-yellow rays excite the green sensation more strongly than the purest green rays, but also excite the red sensation; the yellow rays excite the red

* *Journal of the Franklin Institute*, Jan., 1889.

sensation as intensely as the brightest red rays and the green sensation as intensely as the purest green rays. Maxwell's diagram is a graphic representation of the result of careful photometric measurements of the effect of the spectrum upon these primary sensations.

1, 2, 3 are spectrum colors, which represent primary color sensations, because each excites one primary color sensation exclusively, and *a*, *b*, *c* are curves, showing the relative power of spectrum rays to excite the respective primary sensations.* These conclusions are stated and indorsed in all recent text-books on color, and that eminent physicist, Lord Rayleigh, goes so far as to say that the theory is as well proved as the law of gravitation.

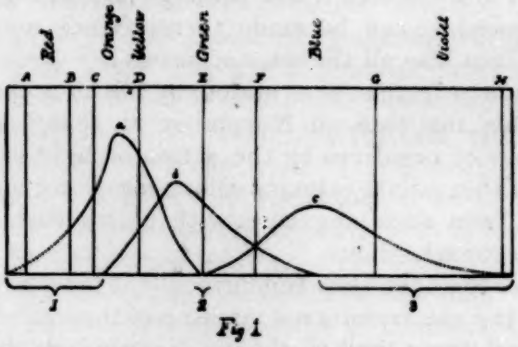


Fig 1

I emphasize this, because another theory of color has recently been proposed, which, although it has been circulated in the newspapers as if it had high scientific indorsement, is really inconsistent with established facts, and cannot, therefore, be true.

The carrying out of my new principle, according to Maxwell's measurements, therefore, involves the production of one negative by the joint action of the red, orange, yellow

* Taken separately, each of these curves is probably correct; but the three do not compound to represent accurately the relative visual intensity of different parts of the spectrum, because the curve of the green sensation is relatively too low, and that of the blue sensation too high; if the curve of the green sensation be drawn as high as that of the red, and the height of the blue curve be reduced one-half, they will compound to represent fairly a photometric measurement of the visual intensity of the spectrum.

and yellow-green rays, in definite proportions, to represent the red sensation; one by the joint action of the orange, yellow, green and green-blue rays, in definite proportions, to represent the green sensation, and one by the joint action of the blue-green, blue and violet rays, in definite proportions, to represent the blue sensation.

Negatives of the required character can be made by exposing a cyanine-stained gelatine-bromide plate through a double screen of chrysoidine-orange and aniline-yellow of suitable intensity for the red sensation, a cyanine-erythrosine gelatine-bromide plate through a screen of aniline-yellow of suitable intensity for the green sensation, and an ordinary gelatine-bromide plate through a double screen of crysophenine-yellow and RRmethyl-violet for the blue sensation. The plates and screens are correct when they will secure negatives of the spectrum showing intensity curves substantially like the curves in Maxwell's diagram. The negatives can also be made on certain makes of ordinary commercial gelatine-bromide plates of the most rapid kind, by the use of quite different color screens for the first two, but only with exposures of from five to fifteen minutes on well-lighted landscapes, aperture of objective *f.* 12.*

In photographing objects in a changing light, landscapes, for instance, it is important that the three sensitive plates be exposed simultaneously; and in order to accomplish this, I devised a triple camera, having three lenses so arranged in connection with reflectors as to bring all the points of view within a one-inch circle. With this camera, the production of sets of negatives of the required character is a simple and easy matter, it being only necessary to insert the plates, raise the flap until the exposure is made, take the

* I claimed that the production of such a set of negatives as I have described represented the application of a new principle in composite heliochromy. Some photographic *littérateurs* were disposed to sneer at such a claim; but after leaving the matter open for discussion for nearly two years, I applied for a United States patent, and, in view of all the evidence, was granted exclusive right to make such sets of negatives.

plates out again, and, when convenient, to develop them together, in the ordinary way.

There are two ways of making the heliochromic pictures from these negatives. The first method does not produce a permanent picture, but a screen projection.

Lantern slides made from the heliochromic negatives and exactly reversing their light and shade must also represent the effect of the object upon the respective color sensations. One lantern positive, when seen by transparency in red light, reproduces the effect of the object upon the primary red sensation. Another, viewed in the same manner by green light, reproduces the effect of the object upon the green sensation. The third, viewed by blue-violet light, reproduces the effect upon the blue sensation. Evidently, the combination of these three images into one must form a reproduction of the object as seen by the eye, correct in form, color, and light and shade. Such a combination is effected by projecting the three pictures with a triple optical lantern, so that they exactly coincide upon the screen. The result is what we have been led to expect.

We have here a true solution of the problem of reproducing the colors of nature in a screen picture, dating from November, 1888. Previous to the publication of my new principle, it was assumed by Cros, Poirée and others, that if the projection method were employed, each picture should be projected by the same kind of rays as those which acted to produce it. In my method, as I have already stated, a picture made by the joint action of red, orange, yellow and yellow-green rays, but chiefly by orange, instead of being projected by a similar mixture of spectrum rays, is projected by red rays only. Similarly, the picture made by orange, yellow, green and green-blue rays is projected by green rays only, and that made by blue-green, blue and violet rays, by blue-violet rays only. That is the true principle, yet nothing of the kind had ever been suggested. The process is capable of giving results which are above criticism, except of that hair-splitting kind which applies also to the ordinary photographic process as a means of reproducing objects which have no color. The most serious objection to this method of solving

the problem is that its only commercial value would lie in its application to the illustration of popular lectures.

Dr. Stolze, who was one of the first to recognize the genuineness of this solution of the problem, doubted if, even in theory, color prints from the same kind of negatives could be made to furnish such a perfect solution. A year ago, I also believed that there were theoretical difficulties in the way of realizing perfect process with color prints. Only recently have I succeeded in showing what relation the colors of the prints must bear to the colors of light used in projection, in order to perform exactly the same function and, under like conditions of illumination, secure equally perfect fulfilment of theoretical requirements.

In the projecting method, we build up the luminous image by adding light to light. White light is produced by the mixture of the three colored lights used for projection, and black by their suppression. But when we carry out the process to produce permanent pictures, the paper which may form the basis of the picture is itself white, and it is the shadows that are built up by the superposition of color prints.

Nevertheless, the color print has exactly the same function to perform as the lantern positive, *i. e.*, to absorb and suppress, by its shading, light affecting one primary color sensation. If we remove our three positives from the lantern, the screen is evenly illuminated with white light. If we then replace the one representing the green sensation, its shadows will absorb the green light, with the result that the screen bears a picture in the complementary color, pink, on a white ground. In the color-print method, we commence with a white surface, which corresponds to the fully illuminated screen, and the shadows of the color print representing the green sensation, when laid upon this surface, absorb the same kind of rays as the shadows of the positive in the lantern, and with the same result, a pink monochrome picture on a white ground. Superposing the other two color prints upon the first one on paper is like inserting the other two positives in the lantern. This explains why the primary sensations are represented by prints having shades

of the complementary (absorbing) color. It is the lights and not the shades of the color prints that represent the effect upon the respective primary color sensation. It is only necessary to use dyes that completely absorb red light but neither green nor blue-violet for the print representing the red sensation, green but neither red nor blue-violet for the green sensation, blue-violet but neither red nor green for the blue sensation, in order to obtain from my negatives a color print heliochrome that exactly fulfils all theoretical requirements, provided that it be examined in the same kind of white light that we obtain in the screen projections, by mixing red, green and blue-violet rays. The dyes mentioned by me in my paper of November 21, 1888 (prussian-blue, aniline-magenta and aniline-yellow), fulfil this requirement, and color-print heliochromes made therewith according to my instructions must, therefore, reproduce all the colors of nature under the conditions of illumination just stated.

We have, then, a theoretically perfect and, at the same time, practicable process of reproducing all the colors of nature in permanent prints from three negatives.

In order to obtain colors that would appear of exactly the right kind and shade in ordinary white light, it would be necessary to use dyes each of which completely absorbed all light affecting the color sensation which it represented, but no other. The colors would then be correct in ordinary white light, but would appear too dark, relatively to the white ground. In order to obtain colors that appear brighter in ordinary white light, dyes may be used which completely absorb only rays that excite chiefly single primary sensations and other rays in due proportion. The dyes proposed by me also fulfil this requirement, so that even in ordinary white light the degradation of a color is insignificant, except in the greens, where it is noticeable.

I have seen some of the results produced by the older processes of composite heliochromy, and others who have also seen them will, I am sure, bear me out when I say that the colors have invariably been not only untrue, but either very dull or else flat and patchy and wanting in the delicate

details and gradations of light and shade which characterize good monochrome photographs. All that showed bright colors resembled nothing so much as cheap chromos. In the composite helichromes by my process, which I show to-night, the colors are, as you can see, as perfect in detail and gradation as the monochrome shades of an ordinary photograph.

According to Captain Abney, the pictures produced by the silver sub-chloride process are "photographs in natural colors." Those which I now show are not so in the same sense, but they are something more and better than that—they are photographs in the colors of nature.

I have already spoken of a class of writers who still believe in the ultimate success of the silver chloride process, or something like it. It is not very surprising that men imbued with such a belief should be displeased with the idea of composite heliochromy, which is something short of a miracle; but the worst of it is, that they will not even take the trouble to make themselves familiar with the subject, and almost every utterance they make in reference to it is calculated to mislead the public and discredit true progress. I do not know how many times such writers have assured the public that composite heliochromy was based upon the same principle as chromo-lithography, and was merely the production of colored pictures by the aid of photography. Even Dr. Eder, a most able photographic chemist, is reported to have assured the representative of a London newspaper that my process was "chromo-lithography, in which photography is only an accessory!" Now, there really is such a thing as photochromy, which is carried out on the same plan as chromo-lithography, but it is no more like composite heliochromy than the Morse system of telegraphy is like telephony. In photochromy, it is only necessary for the photographer to make one negative of the object to be reproduced, and this negative contains a register of form and light and shade only. Composite heliochromy cannot be carried out with less than three negatives, which must contain a register not only of form and light and shade, but of color also. In photochromy, an artist is employed to regulate the

distribution of colors, according to his taste or judgment; in composite heliochromy, it is the light itself which regulates their distribution and combination, automatically, according to fixed and true scientific principles. Photochromy is an art; composite heliochromy, a science.

There are others who do make a distinction between photochromy and composite heliochromy, but whose statements are nevertheless too misleading to have any value. Only a few months ago, there appeared in one of the oldest and most pretentious of the photographic journals, an editorial article upon this subject, in which reference was made to "the three primary colors, red, yellow and blue," and all advance made upon the basis of true theory was discredited. Another, writing for an important periodical, said, "the red, yellow and blue" theory worked well enough in practice, and classed as an "advanced worker" one who had never got beyond that idea in composite heliochromy, or even contributed anything to its development. Dr. H. W. Vogel, taking advantage of the prevailing ignorance, even tried to make the readers of *Anthony's Bulletin* believe that I had claimed as my own something which belonged to him.*

The frequency of such misrepresentation by writers, from whom the public has a right to expect something very different, is my justification for assuming the office of teacher and historian long enough to state the facts, which many people have wished to know, but could not discover by reference to current photographic literature.

In conclusion, for the benefit of those who would like to know why this process is not now in commercial operation, having been perfected in theory three years ago, I will say that, for various reasons, it is not practically available to one whose time is nearly all taken up with a business of a different character, and I do not expect to do much with it until I shall have completed preparations which will justify me in making it my chief occupation. In order to carry out the process in strict accordance with the theoretical requirements, means must be employed not only to secure three negatives and three prints, each of which is correct by

* *Anthony's Bulletin*, 1889, p. 101.

itself, but each must bear also a certain definite relation to the others. A very little over- or under-exposure of any one color print, or a very little too much or too little of the color stuff in the film, will change the shade of delicate colors. Fortunately, there is a simple optical test by which such a defect can be detected without reference to, or knowledge of, the colors of the object photographed; but at present it is difficult to secure such harmony of parts when but little time can be spared to devote to the operation of the process.

Composite heliochromy must always remain a comparatively costly process, when carried out in a manner calculated to yield the finest results, and can most profitably be brought before the public in the form of optical lantern lecture illustrations, not with the triple lantern, but with transparent color-print heliochromes mounted as lantern slides. If the color prints are made by the Woodburytype process, such heliochromic lantern slides, infinitely superior to hand-painted ones, can be made in quantity at a cost not exceeding one dollar each.

ELECTRICITY: ITS PAST, PRESENT AND FUTURE.

BY RALPH W. POPE,
Secretary American Institute of Electrical Engineers.

[*A lecture delivered before the Franklin Institute, December 16, 1889.*]

The lecturer was introduced by Prof. EDWIN J. HOUSTON, and spoke as follows:

MEMBERS OF THE INSTITUTE, LADIES AND GENTLEMEN:

We can have no conception of that period in the physical history of the world when electrical manifestations were unknown. This once mysterious natural agency existed before the creation of man. Electricity, however, direct from the laboratory of nature has not been subdued and impressed into our service, in the same sense that we have for centuries utilized the power of wind; or the force of gravity through the medium of falling water. Natural electricity, as it appears to us in the form of lightning from the clouds,

seems to be wasted energy. We may have learned from our school-books that it clears and purifies the air, yet thousands of our people are compelled to live where it is extremely doubtful if the air they breathe is improved by its influence.

All of the earlier electrical experiments were confined to the domain of frictional or static electricity, a branch of the science which, although interesting and instructive, has been of little or no practical use to mankind. It is the field of applied electricity, its earlier and most important service in the transmission of intelligence which first demands our attention. The original invention by Volta, of the so-called "pile" made public in 1800, was the important step which led up to the electrical triumphs of the nineteenth century. This seems to have been the first discovery of that form of electricity which has since been known as the voltaic or galvanic current. It is not sufficient for our purpose, however, that we should simply be aware of the existence of a force. However interesting it may be as a subject of scientific investigation, its commercial value dates from that period when, in the hands of the inventor, it is led to perform some useful service. The well-known history of the steam engine may be cited as an example of the utilization of a force, the existence of which had been known for ages. So familiar were its manifestations by reason of its intimate association with domestic life, that it probably escaped elaborate scientific investigation until through the genius of James Watt, it received immediate recognition as a most important factor in the industrial field.

The æolipile of Hero, in the history of steam application, may be classed with the galvanometer of Schweigger, invented in 1820. Each was a manifestation of the existence of force, but not in such form as to be at that time commercially applicable. It is interesting to note that the steam turbine utilizing the principle invented by Hero is now coming into successful use for the driving of dynamos, its high speed being especially valuable in this class of work.

Taking the voltaic pile of 1800 and the galvanometer of

1820 we have a combination, which, at first glance, might seem to be the equivalent of the apparatus used for years in submarine cable service, and which is the sole reliance to-day, whenever the atmospheric conditions are unfavorable to the operation of the siphon recorder. Indeed, Ampère in 1820 had already suggested the employment of a battery, lines of wire, and galvanometers, for telegraphic purposes. There was this material defect, however, that existed in the early galvanometer, as well as the later magnetic devices; they were not adapted for operation by electricity, at any great distance. Dr. Ritchie exhibited before the Royal Institution in London, 1830, a model telegraphic apparatus based on the idea of Ampère. In this, a galvanometer and wire being used for each letter of the alphabet, the whole formed too complicated an arrangement for practical use. The fact that none of these early experiments were successful in producing signals at a distance was the stumbling block that baffled every effort of the early electrical inventors. The voltaic pile, made up of alternate discs of silver, zinc, and cloth saturated with acidulated water, although producing a suitable current was not exempt from serious defects, and while it could have been made to serve our purpose for practical telegraphy, it was superseded by the fluid cell in its various forms and combinations, which we may consider as having been virtually perfected before the invention of the electric telegraph. The simplest form of the voltaic cell may be looked upon as the germ from which has sprung up our vast and continually increasing electrical industries.

There was, however, another natural force which had long been known, but which, like electricity, had not been utilized. This was known as magnetism, but in its natural and so-called permanent condition, its earliest use in the form of the magnetic needle, as in the mariner's compass, seems to have been its only important practical application. In the form of load-stone, or magnetic iron ore, this wonderful property of magnetism had been known to mankind for ages. Like electricity, no satisfactory explanation can be given as to what it is, although we are familiar with its

laws and effects. Acting as a continuous force, like that of gravity, it could not be utilized in mechanics, until the relations of electricity and magnetism were investigated, and the wonderful discovery of electro-magnetism was made by Arago in 1820. We are indebted to William Sturgeon for the discovery, in 1825, that inductive magnetic effects could be intensified by forcing an electric current to pass round and round a bar of soft iron, and that this magnetic effect entirely disappeared with the cessation of the current. Electricity and magnetism thus joined forces in the electro-magnet and they are now studied and practised jointly. Returning to the simple voltaic cell, which, as was shown, may be taken as the basis of electrical work, it should be thoroughly understood that what is known as the electric circuit, must in all cases be made complete, in the fields of telegraphy, telephony, electric lighting and power transmission. This peculiarity of electricity is not readily comprehended. Its action may be compared with that of the ordinary belt used in driving machinery, the belt of a sewing machine, for instance. If this belt is continuous, its work may be satisfactorily performed, provided its tension is properly adjusted. If the belt is broken, its continuity must be restored before work can be resumed. So with the electric current, let the circuit be broken, no energy can be transmitted, and, therefore, no work can be done until the circuit is restored. If a belt is slack and slips on the pulleys, we lose energy; and in a similar manner, if a conductor of electricity is not properly insulated, only a portion of the energy is transmitted.

Now, we may also conceive of the electric current acting like a belt in its inductive effects. The belt and pulleys running in one direction brought into contact with another belt and pulleys in such a manner, as to drive them by friction, would cause rotation in the opposite direction. Thus an electric current over one wire will cause an induced current to flow in an adjacent parallel wire, but of an opposite polarity, or in an opposite direction, as is more popularly understood. There is one more important property of electricity and magnetism which it is necessary to

consider, before we are prepared to survey the electrical field as a whole. It is a very simple proposition, that if electricity produced magnetism, and through the action of magnetism we produce motion; that motion applied to a magnet, under similar conditions would produce electricity.

This discovery by Faraday, of what is known as magneto-electricity, was not immediately appreciated in its final bearing upon electrical development as we see it to-day. Its principle was briefly that the movement of a permanent magnet in close proximity to a coil of insulated wire produced in the latter, an electrical impulse. This idea was developed into the magneto-electric machine, which, beyond its utilization, to some extent for medical purposes and electro-plating did not come into immediate use, although it was important as being the foundation of the dynamo of to-day, in which the permanent magnets have been superseded by the electro-magnet. It was discovered by Farmer, Siemens, Wheatstone, and Varley, at about the same time, in 1866, that the residual magnetism in the machine was sufficient without the use of permanent magnets.

The history of electricity is more complete than that of any other similar subject, therefore, we are able to trace the development of various inventions, credit for which, however, has, no doubt, been frequently misapplied. In treating of electricity in the past, we should take it up at that period when, through the medium of the electric telegraph, it served a purpose not previously met by any device known to man—that of communicating intelligible signals instantaneously between points separated by distances of hundreds or even thousands of miles. We have seen that as early as 1820, using the pile of Volta, and the galvanometer of Schweigger, we were on the eve of a great invention, yet it remained for a bright practical mind to overcome that obstacle which proved insurmountable to those who had already conceived the possibility of an electric telegraph.

It is through this gradual evolution of invention that the patient work of the early investigators is too frequently buried beneath the laurels which are placed upon the brow of him whom the world recognizes as the successful inven-

tor. We may honor Morse without disparaging the work of Arago, Sturgeon, or Henry, to whom he was indebted for the electro-magnet, without which his electric telegraph was a mere vision. The struggles of Prof. Morse with poverty, the perseverance with which he labored to secure a trial of his invention, the discouragements he encountered even from his friends are not only familiar to us, but we know it to be too frequently the experience of pioneers who are in the van. It is this very qualification of perseverance, however, that is frequently lacking in the temperament of the deserving inventor, and the field which he planted is reaped by other hands. The crude and cumbersome apparatus, originally used by Morse, has been gradually simplified and perfected, until it stands to-day, and will probably continue to stand, without a rival, for simplicity and effectiveness. The electro-magnet, originally used by Morse, was not adapted for use on circuits of a sufficient length to make his telegraph a commercial success. Through Dr. Gale he learned from Henry's experiments that the strength of the magnet could be increased by winding additional layers of wire upon it. It appears, however, that the same size of copper wire was used, both on the line and for the magnets. This necessitated the use of about 180 pounds of wire, in order to produce the same magnetic effect that we now obtain with about half a pound of fine magnet wire. In order to secure sufficient power to operate his embossing apparatus, he found it necessary to interpose what has since been known as the relay magnet, in which the main circuit was used to repeat the signals on a local circuit, and thus produce the necessary effort to mark the paper with a metal stylus. This ingenious device was a necessary element in the successful operation of the Morse telegraph up to the time when operators through long practice, had begun to read the signals by sound. Although the Morse recording register of the past is practically obsolete in commercial telegraphy, the relay and local battery still remain in service, the power required for the sounder being necessary in order to obtain the best practical results. So long as this feature is retained, and the alphabet of the distinguished inventor

remains in use, the telegraph should be known, as it is universally called to-day, the Morse system.

The more recent inventions which are identified with it, the duplex of Stearns, the quadruplex of Edison, and the multiplex of Delany, are all improvements in the direction of economy; that is they have increased the capacity of each wire, so that a much larger volume of business can be transmitted and the earning capacity of the plant increased accordingly. Each of the additional circuits thus created—phantom wires, as they are termed—requires the attendance of a skilled Morse operator at the transmitting, as well as the receiving station. This class of labor is considered by the companies as highly paid, but, as a matter of fact, the natural intelligence required, and the length of time devoted to attaining proficiency, is certainly entitled to reasonable compensation. The practical success of the Morse system, naturally stimulated the efforts of other inventors, and there was issued, in 1846, a patent to Royal E. House, for an improvement in electric telegraphs. This was a type-printing instrument; and, notwithstanding all that has been accomplished since that time, it must still be considered a masterpiece of ingenuity; and its practical perfection for its intended use, the mechanical skill displayed in making it a serviceable and durable machine for actual work, the House printer, as it was familiarly called, may fairly be considered an extraordinary invention. A successful trial of this machine was made in 1849 on the line between New York and Philadelphia. For speed and accuracy it was unrivalled in its day, but owing to the rapidity of the electrical pulsations essential to its operation, it was not adapted for use on the long circuits which the geographical conditions of the country rendered necessary. It gradually went out of use about 1860, being partially succeeded by the type-printing instrument of David E. Hughes, another beautiful invention introduced in 1857, which at one time seemed likely to largely replace the Morse apparatus. Some of the features of the House and Hughes printers were blended in the combination printer by the late George M. Phelps, who, subsequently invented the motor printer,

now used to a limited extent between New York and Boston, and New York, Philadelphia and Washington.

The Bain system, used by one of the rival telegraph companies in this country, forty years ago, was a chemical telegraph, a dot and line alphabet similar to the Morse being used, the current however, instead of producing a mechanical effect, was forced to pass through a sheet of moving paper, moistened with a chemical solution, upon which the characters appeared in dark blue lines. The only vestige of this invention, which has survived, is the occasional use of the chemically prepared slip in systems similar to the Wheatstone automatic telegraph, by which signals are transmitted at the rate of 500 words per minute, from perforated slips previously prepared. This system is being extensively introduced by the Western Union Telegraph Company, its principal merits being the increased capacity of a wire between cities where a large volume of business is transmitted, and the possibility of dispensing with the highest grade of operators. Messages are copied from the slip by an operator using a typewriter. It will be seen that electricity in the past has found its field in the commercial world as a conveyer of signals, convertible into intelligible communications as rapidly as the most expert penman could write them. Although the electrical action is instantaneous, there are certain conditions which continue to cause occasional delay and annoyance in the transmission of telegrams. The pressure of public opinion as voiced by the press has always been in the direction of lower rates, rather than toward prompt and accurate service, and the natural consequence has been a tendency to economy, which has not always or usually been conducive to the best results.

What electricity may do under proper conditions will, perhaps, be most strikingly shown by a brief reference to its public service for fire-alarm purposes. Nowhere is the value of time more forcibly shown than in the management of the fire telegraphs in our principal cities. Every precaution has been taken to remove all possible chances of failure, and there can be but one opinion as to the import

ant service thus rendered by electricity. The adaptability of the telegraph to this work was soon appreciated, and the active brain of Moses G. Farmer soon devised a practical method of utilizing electricity for this purpose. The simple break-wheel with its fixed characters, revolved by clock-work, was the foundation of the system, but there has been grafted upon it in later years a most elaborate combination of repeaters, gongs and various apparatus, forming altogether a most interesting collection of electrical machinery, which is not so generally appreciated as it should be. In this service we see electricity combined with highly trained men and animals, ever on the alert to aid, in its most important mission, for the safety of life and property. Everything must give way to the flying steamer. The autocratic truck driver must haul to one side, and the district messenger, with his important telegram, must hurry for once, to get out of the way.

In the department of signalling, electricity in the past has had no successful rival. Its various duties in the modern factory, hotel, office and residence, its still closer relation to all of us, through the "stock ticker," the messenger call-box, the burglar alarm, the annunciator, and the telephone, are so familiar that it is merely necessary to mention them to remind you of their value.

Electricity, as utilized by us to-day, has expanded into a much wider field, but one that has long been held by other interests and is viewed as a competitor, an enemy, or an interloper, according to the interests of those whose opinions we listen to. Its use in artificial illumination is perhaps the most important, and certainly the most conspicuous. Its development for this purpose, however, has been seriously retarded, owing to the lack of any sufficiently economical method of producing electricity. The arc lamp may be said to have slumbered from the date of discovery by Sir Humphry Davy, in 1808, until the invention of what is known as the Jablochkoff candle, in 1877. Its commercial possibility was due to the perfection of the dynamo machine by Siemens and Gramme, working independently, in 1871. The arc lamp had, however, been in practical use for light-

house purposes since 1858; the necessary current having been supplied from the magneto generator, based on the principle already referred to.

[*To be continued*]

THE CONTINUOUS GIRDER—VARIABLE MOMENT OF INERTIA—MOMENT AND LOAD COËFFICIENTS—GRAPHICAL METHOD.

BY C. H. LINDENBERGER.

With such a problem as this, all that can be reasonably hoped for to make it practical as well as useful, is that some simple graphical method may be invented in time. The case will then resemble some well-known problems in harbor surveying, or the famous problem of the Howe truss strut diagonal, which no one ever thinks of computing analytically. Most of the graphical methods known at present, however, are quite as complex and tedious as the purely analytic method.

The present article grew out of an attempt of the writer to invent a graphic method that would follow up and explain Prof. Howe's "Continuous Girder," the best, and in fact the only work on this subject in the English language. It was discovered, however, that the objection of graphical complexity and tedium was too strong to be overcome, and that the problem must be re-stated in order to simplify the graphical method. Let the reader judge whether this has been accomplished.

In order to anticipate criticism, it would be, perhaps, better to demonstrate that the results obtained are rigorously identical with those of Prof. Howe, as well as to deduce the new Theorem of Three Moments from the differential equation of the elastic line.

Let us begin by a few definitions:

Let θ equal six times the modulus of elasticity multiplied by the moment of inertia. It will be noticed that the symbol θ is used where Prof. Howe only uses the symbol I, the

latter referring only to the moment of inertia; but it should also be borne in mind that, if E is the modulus of elasticity, then the latter must always be a multiplier of I, except when it disappears as such by cancellation, because it is supposed constant, which is the usual and convenient hypothesis.

ε_v and ε_{v-1} are the horizontal coördinates, whose origin is the r^{th} support, of two points in the r^{th} span, between which θ is constant. The particular value of the latter is denoted by its subscript, which is $(v-1)$. ε_{v-1} is always less than ε_v, and the space between these points may include the whole or any less space over which θ is constant. This is equivalent to giving the option of dividing a portion of the span over which θ is constant, into two or more parts, when it is necessary; and is an option that Prof. Howe does not give, but which evidently would not change the results obtained.

Hence, the subscripts v and $v-1$ do not require any rigid, once for all, system of enumeration of partial lengths to be adopted, and any portion may be subdivided at pleasure.

By Prof. Howe's definition, the value of θ at the right end of the r^{th} span is denoted by θ_r; where the subscript r refers to the span and not to the number of parts. In the same manner, θ_{r-q} would refer to the value of θ at the right end of the $(r-q)^{th}$ span. This is a refinement that is necessary in his analysis, but not in mine.

l_r is the length of the r^{th} span. P_r is a weight on that span, and a_r $(= k_r l_r)$ is its distance from the left support.

Let u_v be any function of ε_v and u_{v-1} a similar function of ε_{v-1}, then take

$$\frac{u_v - u_{v-1}}{\theta_{v-1}} = \Delta u_v \qquad (1)$$

The sign Δ indicates that the difference of the functions is to be divided by θ_{v-1}. The function is always written in terms of ε_v, as follows:

Let $$\frac{\Delta 2\varepsilon_v^3}{l_r^2} = \gamma_v \qquad (2)$$

Let $$\frac{\Delta(3 l_r \varepsilon_v^2 - 2 \varepsilon_v^3)}{l_r^2} = \lambda_v \tag{3}$$

Let $$\frac{\Delta(6 l_r^2 \varepsilon_v - 6 l_r \varepsilon_v^2 + 2 \varepsilon_v^3)}{l_r^2} = \tau_v \tag{4}$$

The ideas expressed by these last three symbols will become apparent when the graphical method is reached. To prevent misapprehension and show the conciseness of the symbol expand the definition of λ_v, thus

$$\lambda_v = \frac{(3 l_r \varepsilon_v^2 - 2 \varepsilon_v^3) - (3 l_r \varepsilon^2{}_{v-1} - 2 \varepsilon^3{}_{v-1})}{l_r^2 \theta_{v-1}} \tag{5}$$

Suppose there were a number of successive differences of the function u, each divided by its appropriate value of θ, to be added; the subscript of the first being $n - 1$ and of the last being m then the series would read as follows:

$$\frac{u_n - u_{n-1}}{\theta_{n-1}} + \frac{u_{n+1} - u_n}{\theta_n} + \text{etc.},$$

$$+ \frac{u_{m-1} - u_{m-2}}{\theta_{m-2}} + \frac{u_m - u_{m-1}}{\theta_{m-1}}$$

Then by inspection we may write

$$\sum_{v-m}^{v-n} \frac{u_v - u_{v-1}}{\theta_{v-1}} = -\frac{u_{n-1}}{\theta_{n-1}} + \frac{u_m}{\theta_{m-1}} + \sum_{v-m-1}^{v-n} u_v \left(\frac{1}{\theta_{v-1}} - \frac{1}{\theta_v}\right) \tag{6}$$

In using this equation, to transform those of Prof. Howe, we will in general have $u_{n-1} = 0$ and $u_m =$ value of function when $\varepsilon_v = l_r$. When, however, u is such a function that it equals a variable plus a constant, then evidently the constant part is cancelled by subtraction in the expression $u_v - u_{v-1}$.

It might then be omitted in the right-hand member, but then we would not have $u_{n-1} = 0$ if it were $= 0$ when the constant part was left in. Notice also that θ_{m-1} will have the value θ_r.

Make

$$\sum_{l_r}^{0} \lambda_v = L_r; \quad \sum_{l_r}^{0} \gamma_v = G_r; \quad \sum_{l_r}^{0} \tau_v = T_r \tag{7}$$

The space over which the summation is taken is shown by the limits. Here it includes the whole span.

M is the moment at the support shown by its subscript.

In Prof. Howe's analysis the ε that has the subscript l_r is the last value but one of ε, or in other words, the space between the points whose ordinates are $\varepsilon_v = l_r$ and $\varepsilon_v = \varepsilon_1$ is the extreme right end space in the span, and between these points θ has the value θ_r. A very confusing refinement necessary in his analysis but not in mine.

Divide Prof. Howe's equation (55) by $\theta_r\,\theta_{r-1}$, and we have the following formula. (Some misprints corrected here and elsewhere.)

$$M_{r-1}\frac{(l_{r-1}+F'_{r-1})}{\theta_{r-1}}+M_r\left\{2\frac{(l_r+F_r)}{\theta_r}+2\frac{(l_{r-1}+F''_{r-1})}{\theta_{r-1}}\right\}$$

$$+M_{r+1}\left(\frac{l_r+F_r'}{\theta}\right)=-\left(\frac{h_r-h_{r-1}}{l_{r-1}}+\frac{h_r-h_{r+1}}{l_r}\right)$$

$$-\frac{(\Sigma P_r\,l_r^2(2\,k_r-3\,k_r^2+k_r^3)+F_r'\,\Sigma\,P_r l_r(1-k_r)-H_r)}{\theta_r}$$

$$-\frac{(\Sigma P_{r-1}P_{r-1}(k_{r-1}-k^3{}_{r-1})+2\,F''_{r-1}\,\Sigma\,P_{r-1}l_{r-1}(1-k_{r-1})+H'_{r-1})}{\theta_{r-1}} \tag{8}$$

Prof. Howe considers moments causing compression on upper chord as positive.

He gives definitions as follows (substituting θ for I, which does not change the meaning):

$$\frac{F_r}{\theta_r}=\sum_{v=l_r}^{v=1}\left(\frac{1}{\theta_{v-1}}-\frac{1}{\theta_v}\right)\left(\frac{3\,l_r^2\,\varepsilon_v-3\,l_r\,\varepsilon_v^2+\varepsilon_v^3}{l_r^2}\right) \tag{9}$$

$$\frac{F_r'}{\theta_r}=\sum_{v=l_r}^{v=1}\left(\frac{1}{\theta_{v-1}}-\frac{1}{\theta_v}\right)\left(\frac{3\,l_r\,\varepsilon_v^2-2\,\varepsilon_v^3}{l_r^2}\right) \tag{10}$$

$$\frac{F_r''}{\theta_r}=\sum_{v=l_r}^{v=1}\left(\frac{1}{\theta_{v-1}}-\frac{1}{\theta_v}\right)\frac{\varepsilon_v^3}{l_r^2} \tag{11}$$

Comparing these equations with equations (2), (3), (4), (6) and (7), it becomes evident that

$$2\left(\frac{l_r + F_r}{\theta_r}\right) = T_r \tag{12}$$

$$\frac{l_r + F_r'}{\theta_r} = L_r \tag{13}$$

$$\frac{2\,(l_r + F_r'')}{\theta_r} = G_r \tag{14}$$

Prof. Howe also gives definitions of H_r and H_r', but in too general a form to be easily apprehended, since it is for a number of weights scattered all over the span.

It is better to consider a single weight only, since the final value of the quantities is the sum of their partial values. A condition that he gives in his analysis is that for each weight $\varepsilon_v > a_r$. We might, however, have $\varepsilon_v = a_r$ to begin with, without changing the result, since in that case $\varepsilon_v - a_r = 0$.

The modified definitions which, nevertheless, express his idea, are as follows:

$$\frac{H_r}{\theta_r} = \sum_{v=l_r}^{\varepsilon_v = a_r}\left(\frac{1}{\theta_{v-1}} - \frac{1}{\theta_v}\right)\left(\frac{P_r\,(\varepsilon_v - a_r)^3}{l_r} + \frac{3\,(l_r - \varepsilon_v)}{l_r}\,P_r\,(\varepsilon_v - a_r)^2\right) \tag{15}$$

$$\frac{H_r'}{\theta_r} = \sum_{v=l_r}^{\varepsilon_v = a_r}\left(\frac{1}{\theta_{v-1}} - \frac{1}{\theta_v}\right)\left(\frac{P_r\,(\varepsilon_v - a_r)^3}{l_r} - \frac{3\,\varepsilon_v}{l_r}\,P_r\,(\varepsilon_v - a_r)^2\right) \tag{16}$$

Now

$$\frac{(\varepsilon_v - a_r)^3 + 3\,(l_r - \varepsilon_v)\,(\varepsilon_v - a_r)^2}{l_r}$$

$$= \frac{3\,l_r\,\varepsilon_v^2 - 6\,l_r\,\varepsilon_v\,a_r + 3\,l_r\,a_r^2 - 2\,\varepsilon_v^3 + 3\,a_r\,\varepsilon_v^2 - a_r^3}{l_r}$$

$$= l_r\left(\frac{3\,l_r\,\varepsilon_v^2 - 2\,\varepsilon_v^3}{l_r^2}\right) - a_r\left(\frac{3\,l_r\,\varepsilon_v^2 - 2\,\varepsilon_v^3}{l_r^2}\right)$$

$$- a_r\left(\frac{6\,l_r^2\,\varepsilon_v - 6\,l_r\,\varepsilon_v^2 + 2\,\varepsilon_v^3}{l_r^2}\right) + \frac{3\,l_r\,a_r^2 - a_r^3}{l_r}$$

Let

$$\sum_{l_r}^{a_r} \left((l_r - a_r)\,\lambda_v - a_r\,\tau_v \right) = V_r \qquad (17)$$

Then by comparison with equation (6)

$$P_r\,V_r = \frac{P_r\,l_r^2\,(1 - k_r^3)}{\theta_r} + \frac{H_r}{\theta_r} \qquad (18)$$

The constant quantity

$$\frac{(3\,l_r\,a_r^2 - a_r^3)}{l_r}$$

disappears by subtraction.

Also

$$\frac{(\varepsilon_v - a_r)^3 - 3\,\varepsilon_v\,(\varepsilon_v - a_r)^2}{l_r} = \frac{-2\,\varepsilon_v^3 + 3\,a_r\,\varepsilon_v^2 - a_r^3}{l_r}$$

$$= -\,l_r\left(\frac{2\,\varepsilon_v^3}{l_r^2}\right) + a_r\left(\frac{2\,\varepsilon_v^3}{l_r^2}\right) + a_r\left(\frac{3\,l_r\,\varepsilon_v^2 - 2\,\varepsilon_v^3}{l_r^2}\right) - \frac{a_r^3}{l_r}$$

Let

$$V_r' = \sum_{l_r}^{a_r} \left((l_r - a_r)\,\gamma_v - a_r\,\lambda_v \right) \qquad (19)$$

Then by comparison with equation (6)

$$P_r\,V_r' = \frac{P_r\,l_r^2\,(2 - 3\,k_r + k_r^3)}{\theta_r} - \frac{H_r'}{\theta_r} \qquad (20)$$

The constant quantity

$$\left(-\frac{a_r^3}{l_r}\right)$$

disappearing as before.

Paying attention to the fact that $(1 - k_r) - (1 - k_r)^3 = 2\,k_r - 3\,k_r^2 + k_r^3$ and $2\,(1 - k_r) - (2 - 3\,k_r + k_r^3) = k_r - k_r^3$, we have

$$P_r\Big((l_r - a_r)\,L_r - V_r \Big)$$

$$= \frac{P_r\,l_r^2\,(2\,k_r - 3\,k_r^2 + k_r^3) + P_r\,F_r'\,l_r\,(1 - k_r) - H_r}{\theta_r} \qquad (21)$$

$$P_r\Big((l_r - a_r)\,G_r - V_r' \Big)$$

$$= \frac{P_r\,l_r^2\,(k_r - k_r^3) + 2\,P_r\,F_r''\,l_r\,(1 - k_r) + H_r'}{\theta_r} \qquad (22)$$

Put

$$(l_r - a_r)\, L_r - V_r = \frac{A_r}{P_r} \tag{21a}$$

and

$$(l_r - a_r)\, G_r - V_r' = \frac{B_r}{P_r} \tag{22a}$$

then the most convenient formulæ, both for analytical and graphical calculation, are

$$(l_r - a_r) \overset{a_r}{\underset{a_r}{\Sigma^{\circ}}} \lambda_v + a_r \overset{a_r}{\underset{l_r}{\Sigma'}} \tau_v = \frac{A_r}{P_r} \tag{21b}$$

$$(l_r - a_r) \overset{a_r}{\underset{a_r}{\Sigma^{\circ}}} \gamma_v + a_r \overset{a_r}{\underset{l_r}{\Sigma'}} \lambda_v = \frac{B_r}{P_r} \tag{22b}$$

These quantities are calculated by interpolating a term as if the part on which θ is constant on which the weight stands were divided into two parts. Thus, for instance, a table for G_r would contain the quantities

$$2\left(\frac{\varepsilon_v{}^3 - \varepsilon^3{}_{v-1}}{\theta_{v-1}\, l_r^2}\right)$$

Let there be a point whose ordinate is ε_p where the value of θ changes on the left. Then to the sum of the tabular values from the left support up to this point is to be added

$$2\left(\frac{a_r{}^3 - \varepsilon_p{}^3}{\theta_p\, l_r^2}\right)$$

and we have the quantity

$$\overset{a_r}{\underset{a_r}{\Sigma^{\circ}}} \gamma_v$$

Collecting results and comparing with our equation (8) or Prof. Howe's equation (55), we have for the *theorem of three moments*

$$M_{r-1}\, L_{r-1} + M_r\,(T_r + G_{r-1}) + M_{r+1}\, L_r = -(A_r + B_{r-1}) - \left(\frac{h_r - h_{r-1}}{l_{r-1}} + \frac{h_r - h_{r+1}}{l_r}\right) \tag{23}$$

in which moments are positive causing compression on upper

chord, and the h^s are positive upwards. To have the moments positive causing tension on upper chord change the sign of the load terms and consider the h^s as positive downwards.

It is a little more convenient in a graphical calculation to consider the latter to be the case, and is the method pursued by Prof. Merriman, which I shall pursue in deducing this theorem of three moments from the differential equation of the elastic line.

Let M_x be the moment at a point in the span, μ the vertical, x the horizontal coördinate of the elastic line, and θ_x the value of θ at this point. The latter corresponds to θ_{v-1}.

Then the equation may be written

$$\frac{d^2\,\mu}{d\,x^2} = \frac{6\,M_x}{\theta_x} \tag{24}$$

But from Merriman's "Continuous Bridges," we have

$$M_x = M_r - S_r\,x + P_r\,(x - a_r)$$

and

$$S_r = \frac{M_r - M_{r+1}}{l_r} + P_r\,(1 - k_r)$$

whence

$$M_x = M_r\,\frac{(l_r - x)}{l_r} + \frac{M_{r+1}\,x}{l_r} - P_r\,(1 - k_r)\,x + P_r\,(x - a_r)$$

It should be remarked that the quantity $P_r\,(x - a_r)$ belongs to the expression for the moment at a point on the right-hand side of the weight and that this term or the integrals or other results consequent upon it are to be omitted for points on the left side. This statement is intended to save a useless repetition of formulæ.

Substitute the value of M_x in equation (24) and integrate there results

$$\frac{d\,\mu}{d\,x} = \frac{M_r\left(\frac{6\,l_r\,x - 3\,x^2}{l_r}\right) + M_{r+1}\frac{3\,x^2}{l_r} - 3\,P_r(1-k_r)\,x^2 + 3\,P(x-a_r)^2}{\theta_x} + C_v \tag{25}$$

Now for

$$x = \varepsilon_v, \frac{d\mu}{dx} = t_v$$

and for

$$x = \varepsilon_{v-1}, \frac{d\mu}{dx} = t_{v-1}$$

Taking these as limits and using the symbol Δ as explained in equation (1), we have

$$t_v - t_{v-1} = M_r \Delta \frac{(6 l_r^2 \varepsilon_v - 3 l_r \varepsilon_v^2)}{l_r^2} + M_{r+1} \Delta \frac{3 l_r \varepsilon_v^2}{l_r^2}$$

$$- P_r (1 - k_r) \Delta 3 \varepsilon_v^2 + P_r \Delta (3 \varepsilon_v^2 - 6 \varepsilon_v a_r)$$

and from equations (2), (3) and (4)

$$t_v - t_{v-1} = M_r (\lambda_v + \tau_v) + M_{r+1} (\lambda_v + \gamma_v) - P_r l_r (1 - k_r) (\lambda_v + \gamma_v)$$

$$+ P_r \Big(l_r (\lambda_v + \gamma_v) - a_r (2 \lambda_v + \gamma_v + \tau_v) \Big) \qquad (26)$$

$$l_r (\lambda_v + \gamma_v) - a_r (2 \lambda_v + \gamma_v + \tau_v) = (l_r - a_r) (\lambda_v + \gamma_v) - a_r (\lambda_v + \tau_v)$$

Take the sum of the differences of the tangents for the whole span, remembering that the elastic line consists of a series of shorter lines, and at the points of junction they have the same tangent and ordinate. At the left support the tangent is t_r and at the right it is t_{r+1}. All the intermediate tangents will disappear and we get

$$t_{r+1} - t_r = M_r (L_r + T_r) + M_{r+1} (L_r + G_r)$$

$$- P_r l_r (1 - k_r) (L_r + G_r) + P_r (V_r + V_r') \qquad (27)$$

See equations (17) and (19).

Integrating equation (25)

$$\mu = \frac{M_r \left(\frac{3 l_r x^2 - x^3}{l_r} \right) + M_{r+1} \frac{x^3}{l_r} - P_r (1 - k_r) x^3 + P_r (x - a_r)^3}{\theta_x}$$

$$+ C_v x + C_v'$$

For

$$x = \varepsilon_v, \mu = \mu_v, \text{ for } x = \varepsilon_{v-1}, \mu = \mu_{v-1}$$

Also for the constant C_v we have two equal values.

The first is

$$t_v + \left\{ \begin{array}{l} - M_r \left(\dfrac{6\, l_r\, \varepsilon_v - 3\, \varepsilon_v^2}{l_r} \right) - M_{r+1} \dfrac{3\, \varepsilon_v^2}{l_r} \\ + P_r (1 - k_r)\, 3\, \varepsilon_v^2 - P_r (3\, \varepsilon_v^2 - 6\, \varepsilon_v\, a + 3\, a_r^2) \end{array} \right\} \div \theta_x$$

and the second is similar to this except that the subscript of ε and t becomes $v-1$ instead of v.

Taking the integral between these limits we have therefore

$$\mu_v - \mu_{v-1} = - M_r \int \left(\frac{3\, l_r\, \varepsilon_v^2 - 2\, \varepsilon_v^3}{l_r} \right) - M_{r+1} \int \frac{2\, \varepsilon_v^3}{l_r}$$

$$+ P_r (1 - k_r) \int 2\, \varepsilon_v^3 + P_r \int \left\{ (\varepsilon_v - a_r)^3 - 3\, \varepsilon_v\, (\varepsilon_v - a_r)^2 \right\}$$

$$+ t_v\, \varepsilon_v - t_{v-1}\, \varepsilon_{v-1}$$

and from equations (2), (3) and (4) this becomes

$$\mu_v - \mu_{v-1} = - l_r \left(M_r \lambda_v + M_{r+1}\, \gamma_v - P_r\, l_r (1 - k_r)\, \gamma_v \right.$$

$$\left. + P_r \left((l_r - a_r)\, \gamma_v - a_r\, \lambda_v \right) \right) + t_v\, \varepsilon_v - t_{v-1}\, \varepsilon_{v-1}$$

Take the sum of the differences of the vertical ordinates for the whole span and then divide by l_r. For the left support the vertical ordinate is h_r and for the right support it is h_{r+1}, and we have

$$\frac{h_{r+1} - h_r}{l_r} + M_r\, L_r + M_{r+1}\, G_r - P_r\, l_r (1 - k_r)\, G_r + P_r\, V_r' = t_{r+1} \tag{28}$$

Reduce the subscripts by unity and we get

$$\frac{h_r - h_{r-1}}{l_{r-1}} + M_{r-1}\, L_{r-1} + M_r\, G_{r-1} - P_{r-1}\, l_{r-1} (1 - k_{r-1})\, G_{r-1}$$

$$+ P_{r-1}\, V'_{r-1} = t_r \tag{29}$$

Subtract equation (28) from equation (27), and then to the remainder add equation (29), and we get

$$\left(\begin{array}{l} \dfrac{h_r - h_{r+1}}{l_r} + \dfrac{h_r - h_{r-1}}{l_{r-1}} + M_{r-1}\, L_{r-1} + M_r\, (T_r + G_{r-1}) \\ + M_{r+1}\, L_r - P_r \left(l_r (1 - k_r)\, L_r - V_r \right) \\ - P_{r-1} \left(l_{r-1}\, (1 - k_{r-1})\, G_{r-1} - V'_{r-1} \right) \end{array} \right) = 0 \tag{30}$$

This, as will be seen, is the same as equation (23), except that moments causing tension on upper chord are considered as positive.

It is evident from the preceding analysis that if we had considered the modulus of elasticity as variable, it would not affect the form of the formulæ. It would, however, affect the strains considerably, but θ being a product, must of course always be constant over any short portion of the span.

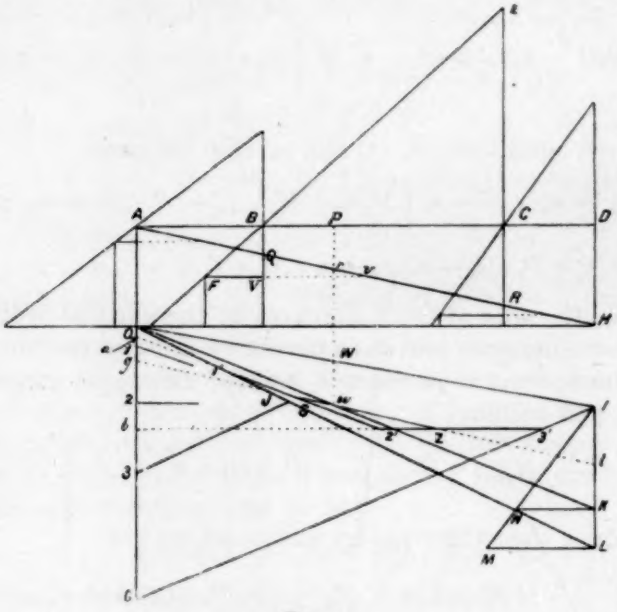


FIG. 1.

The graphical method employed to obtain the necessary quantities L_r G_r T_r A_r and B_r remains to be explained. It should be remarked that in practice *Figs. 1* and *2* are constructed on the same paper, but are separated here for the sake of clearness and distinctness in the demonstration. A_r and B_r are found for a single position of a load, but it will soon be apparent that these quantities can be found for a single load for an infinite number of positions *from the same diagram*, and this is not possible by any other graphical method that I know of.

I will explain *Fig. 1* by giving the construction first and the proof afterwards.

$\overline{A\,D}$ represents the span, and the values of θ change at B and C. Hence $\overline{A\,B} = \varepsilon_{v-1}$ and $\overline{A\,C} = \varepsilon_v$ for the calculation of L_r, T_r and G_r. The vertical $\overline{A\,O} = \overline{D\,H}$ is an arbitrary length of line $= m$. Complete the parallelogram and draw the diagonal $\overline{A\,H}$. At C erect the vertical

$$\overline{E\,C} = \frac{\theta_{v-1}\, l_r}{6\,\theta'}$$

where θ_{v-1} is the value of θ between B and C, and θ' is an arbitrary value of θ. When, as is customary, E the modulus of elasticity is assumed to be constant, make

$$\frac{l_r}{6\,\theta'} =$$

some easy fraction, such as

$$\frac{1}{10\,E'} \qquad \frac{1}{50\,E'}$$

or the like. $(E' = 6\,E)$.

Take

$$\overline{B\,V} = m\left(\frac{\varepsilon_v + \varepsilon_{v-1}}{2\,l_r}\right)$$

which is done by bisecting $\overline{Q\,R}$ at v and drawing the horizontal line $\overline{V\,v}$. This line produced intersects the line $B\,E$ produced at F. Consider $\overline{F\,V}$ as if it were a vertical force $= q_v$ applied at the centre of gravity of the area $B\,C\,R\,Q$. Make the same construction for all the other areas within the triangle $A\,D\,H$ and between whose limits the value of θ is constant. With these lines as forces construct the force polygon O–1–2–3–J with an assumed pole distance $= J$, and then construct the equilibrium polygon O-1′–2′–3′–I. Produce the end sides $\overline{O\text{–}1'}$ and $\overline{3'\text{–}I}$ to the verticals through the supports at the ends of the span, then

$$\overline{I\,L} = \frac{m\,t'}{J}\,L_r$$

and

$$\overline{O\,G} = \frac{m\,\theta'}{J}\,G_r$$

The values of L_r and G_r are now easily computed, since m, θ' and J have been arbitrarily assumed.

The proof is as follows:

$$\overline{F\,V} = q_v = \frac{\overline{B\,V} \times \overline{B\,C}}{\overline{E\,C}} = \frac{3\,m\,\theta'\,(\varepsilon_v^2 - \varepsilon^2{}_{v-1})}{\theta_{v-1}\,l_r^2} = \frac{m\,\theta'}{l_r}\,(\lambda_v + \gamma_v) \tag{31}$$

Now since

$$\overline{B\,Q} = \frac{m\,\varepsilon_{v-1}}{l_r}$$

and

$$\overline{C\,R} = m\,\frac{\varepsilon_v}{l_r}$$

then

$$q_v = \frac{6\,\theta'}{\theta_{v-1}\,l_r}\,(\text{area}\,\overline{B\,C\,R\,Q}) \tag{32}$$

The moment of the triangle $\overline{A\,C\,R}$ with reference to the left support is

$$\frac{m\,\varepsilon_v^3}{2\,l_r} \times \frac{2}{3}\,\varepsilon_v$$

and, therefore, the moment of the area $\overline{B\,C\,R\,Q}$ is

$$\frac{m\,l_r}{6}\left(\frac{2\,\varepsilon_v^3}{l_r^2} - \frac{2\,\varepsilon^3{}_{v-1}}{l_r^2}\right) = \frac{m\,l_r\,\theta_{v-1}}{6}\,\gamma_v$$

Let X_v be the distance of the centre of gravity of this area from the left support. Multiply both sides of the preceding equation by

$$\frac{X_v}{J}$$

and for the moment of this area put its value as found above, and we have

$$\frac{q_v\,X_v}{J} = \frac{m\,\theta'}{J}\,\gamma_v \tag{33}$$

This is represented by the length of a line on the vertical through the left end of the span, and the method is a

familiar one to those who use the equilibrium polygon, but it would be perhaps better to explain this here. In *Fig. 1*, in the force polygon, $\overline{1 \cdot 2} = \overline{F\ V} = q_v$. Prolong the sides of the equilibrium polygon $\overline{1' \cdot 2'}$ to a and $\overline{2'\ 3'}$ to b, the latter being points on the vertical through the left support. Then $\overline{a\ b} : \overline{1\text{–}2} :: X_v : J$ or

$$\overline{a\ b} = \frac{q_v\ X_v}{J}$$

Hence

$$\overline{O\ G} = \Sigma \frac{q_v\ X_v}{J} = \frac{m\ \theta'}{J} G_r$$

The moment of the triangle $A\ C\ R$ with reference to the right support is

$$\frac{m\ \varepsilon_v^2}{2\ l_r}\left(l_r - \frac{2}{3}\varepsilon_v\right)$$

whence we easily deduce as before that the moment of the area $B\ C\ R\ Q$ with reference to same support is

$$\frac{m\ l_r\ \theta_{v-1}}{6}\lambda_v$$

Multiply both sides of equation (32) by

$$\frac{l_r - X_v}{J}$$

we obtain in a similar manner

$$\frac{q_v\ (l_r - X_v)^4}{J} = \frac{m\ \theta'}{J}\lambda_v \qquad (34)$$

Hence

$$I\ L = \Sigma \frac{q_v\ (l_r - X_v)}{J} = \frac{m\ \theta'}{J} L_r$$

The next is the calculation of B_r in *Fig. 1*. The weight is located at p, so that $\overline{A\ p} = a_r$ and θ is constant (as was said before) between B and C. Suppose the area $\overline{B\ C\ R\ Q}$ divided into two areas by the vertical through p. The centre of gravity of the partial area $\overline{B\ p\ r\ Q}$ is in a vertical through s, and that of the area $\overline{p\ C\ R\ r}$ is in a vertical

through z. Now, since the moment of the whole area is the resultant of those of the partial areas, it is evident that $O \cdot 1' \cdot s \cdot z \cdot 3' \cdot I$ is the equilibrium polygon we would have had if we had originally divided the area and made the graphical construction previously shown.

Draw $\overline{s\ z}$ and produce it to the verticals through the supports, g being the left intersection and l the right. Then by previous analysis

$$\overline{O\,g} = \frac{m\,\theta'}{J} \sum_{a_r}^{0} \gamma_v$$

$$\overline{I\,l} = \frac{m\,\theta'}{J} \sum_{l_r}^{a_r} \lambda_v$$

and by similar triangles

$$\overline{l\,K} = \overline{O\,g}\ \frac{l_r - a_r}{a_r} = \frac{m\,\theta'}{J}\left(\frac{l_r - a_r}{a_r}\right) \sum_{a_r}^{0} \gamma_v$$

and, therefore,

$$\overline{I\,K} = \frac{(l_r - a_r) \sum_{a_r}^{0} \gamma_v + a_r \sum_{l_r}^{a_r} \lambda_v}{a_r} \times \frac{m\,\theta'}{J} = \frac{m\,\theta'}{J} \cdot \frac{B_r}{P_r a_r}$$

$$\overline{W\omega} = \overline{I\,K}\,\frac{a_r}{l_r} = \frac{m\,\theta'}{J\,l_r} \cdot \frac{B_r}{P_r} \qquad (Fig.\ 1.)$$

The value of B_r is now known, but it is sometimes more convenient to calculate the moments at the supports directly by a graphical construction, using the "fixed points." In that case we will want

$$\frac{B_r}{L_r\,l_r}$$

Now, since

$$\overline{I\,L} = \frac{m\,\theta'}{J} L_r$$

draw

$$\overline{L\,M} = \frac{P_r\,a_r}{l_r}$$

to as large a scale as convenient; then by similar triangles

$$\overline{NK} = \frac{\overline{LM} \times \overline{IK}}{\overline{IL}} = \frac{B_r}{L_r l_r}$$

The explanation of *Fig. 2* is now easier, as it is governed by the same theory, but the construction resembles that made

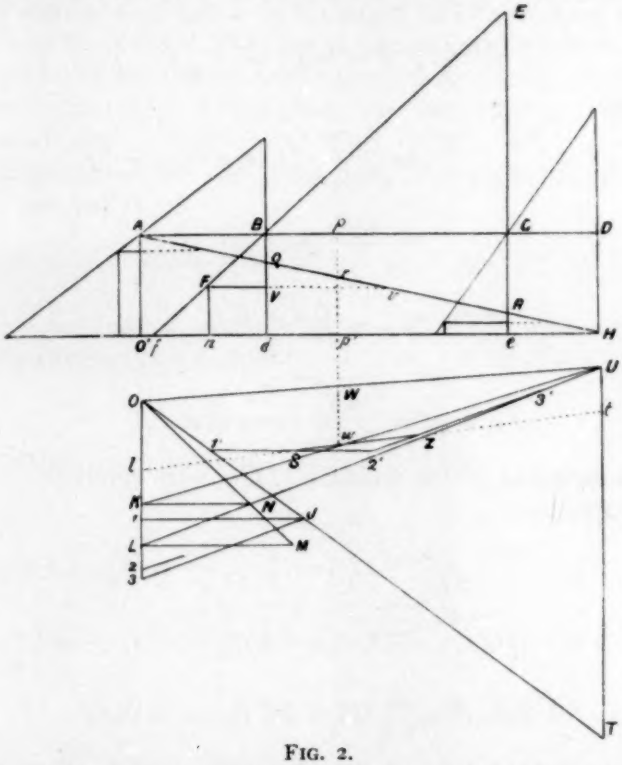


FIG. 2.

as if *Fig. 1* were reversed—that is, as if horizontal ordinates were positive to the left instead of to the right. Thus. $\overline{f\,n}$ is considered to be the vertical force, and it is supposed to be applied at the centre of gravity of the area $\overline{Q\,R\,\varepsilon\,d}$. Areas such as this, between whose limits θ is constant, are contained within the triangle $\overline{A\,H\,O}$.

The force polygon is $\overline{O\text{–}1\text{–}2\text{–}3\text{–}J}$ and the equilibrium polygon is $\overline{O\text{–}1'\text{–}2'\text{–}3'\text{–}U}$.

The construction will be plainer when the proof is given that

$$\overline{O\,L} = \frac{m\,\theta'}{J}\,L_r,$$

and

$$\overline{U\,T} = \frac{m\,\theta'}{J}\,T_r,$$

$$\overline{F\,n} = \overline{V\,d} = m\left(1 - \frac{\varepsilon_v + \varepsilon_{v-1}}{2\,l_r}\right)$$

whence

$$\overline{f\,n} = q_v' = \frac{\overline{F\,n} \times \overline{BC}}{E\,C} = \frac{m\,\theta'}{\theta_{v-1}\,l_r}\left(6\,(\varepsilon_v - \varepsilon_{v-1}) - \frac{3\,(\varepsilon_v^2 - \varepsilon^2_{v-1})}{l_r}\right)$$

$$= \frac{m\,\theta'}{l_r}\,(\lambda_v + \tau_v)$$

also

$$\overline{F\,n} = \frac{\overline{Q\,d} + \overline{R\,\varepsilon}}{2}$$

whence

$$\overline{f\,n} = q_v' = \frac{6\,\theta'}{\theta_{v-1}\,l_r}\,(\text{area } \overline{Q\,R\,\varepsilon\,d}) \qquad (35)$$

The moment of the triangle $\overline{Q\,H\,d}$, with reference to the left support is

$$\frac{m\,(l_r - \varepsilon_{v-1})^2}{2\,l_r}\left(l_r - \frac{2}{3}\,(l_r - \varepsilon_{v-1})\right)$$

$$= \frac{m}{6\,l_r}\,(3\,l_r^3 - 6\,l_r^2\,\varepsilon_{v-1} + 3\,l_r\,\varepsilon^2_{v-1} - 2\,l_r^3 + 6\,l_r^2\,\varepsilon_{v-1} - 6\,l_r\,\varepsilon^2_{v-1}$$

$$+ 2\,\varepsilon^3_{v-1}) = \frac{m}{6\,l_r}\,(l_r^3 - 3\,l_r\,\varepsilon^2_{v-1} + 2\,\varepsilon^3_{v-1})$$

Hence the moment of the area $\overline{Q\,R\,\varepsilon\,d}$, with reference to the same support is

$$= \frac{m\,l_r\,\theta_{v-1}\,\lambda_v}{6}$$

Let X_v' be the distance of the centre of gravity of this area from the left support. Multiply both sides of equation (35) by

$$\frac{X_v'}{J}$$

and we have

$$\frac{q_v' X_v'}{J} = \frac{m \theta'}{J} \lambda_v \qquad (36)$$

Hence

$$\overline{O L} = \Sigma \frac{q_v' X_v'}{J} = \frac{m \theta'}{J} L_r$$

Of course this value of L_r ought to be the same as that obtained from *Fig. 1*. Owing to unpreventable errors it will seldom be exactly the same but any discrepancy shows that there is error and if too great the construction must be repeated.

The moment of the triangle $\overline{Q H d}$ with reference to right support is

$$\frac{m (l_r - \varepsilon_{v-1})^2}{2 l_r} \times \frac{2}{3} (l_r - \varepsilon_{v-1})$$

whence we deduce moment of area $\overline{Q R \varepsilon d}$ with reference to this support is

$$\frac{m l_r \theta_{v-1} \tau_v}{6}$$

Multiply both sides of equation (35) by

$$\frac{l_r - X_v'}{J}$$

and we have

$$q_v' \frac{(l_r - X_v')}{J} = \frac{m \theta'}{J} \tau_v \qquad (37)$$

Hence

$$\overline{U T} = \Sigma \frac{q_v' (l_r - X_v')}{J} = \frac{m \theta'}{J} T_r$$

The value of A_r is found in a way much similar to the way B_r is found in *Fig. 1*. s is in the vertical through the centre of gravity of the partial area $\overline{Q r p' d}$, and z in that of the area $r R \varepsilon p'$. Hence O–1′–s–z–3′–U is the equilibrium polygon we would have obtained had we divided the area $Q R \varepsilon d$ in the first place.

$$\overline{O l} = \frac{m \theta'}{J} \sum_{a_r}^{0} \lambda_v$$

$$\overline{U\,t} = \frac{m\,\theta'}{J} \sum_{l_r}^{a_r} \tau_v$$

and by similar triangles

$$\overline{l\,K} = \overline{U\,t}\ \frac{a_r}{l_r - a_r} = \frac{m\,\theta'}{J}\ \frac{a_r}{l_r - a_r} \sum_{l_r}^{a_r} \tau_v$$

whence

$$\overline{OK} = \frac{(l_r - a_r) \sum^{o} \lambda_v + a_r \sum_{l_r}^{a_r} \tau_v}{\frac{a_r}{l_r - a_r}} \times \frac{m\,\theta'}{J}$$

$$= \frac{m\,\theta'}{J}\ \frac{A_r}{P_r\,(l_r - a_r)}$$

$$\overline{W\,\omega}\quad (Fig.\ 2) = \overline{O\,K}\ \frac{l_r - a_r}{l_r} = \frac{m\,\theta'}{J\,l_r}\ \frac{A_r}{P_r}$$

Let

$$\overline{L\,M} = \frac{P_r\,(l_r - a_r)}{l_r}$$

then

$$N\,K = \frac{\overline{L\,M}\ \cdot\ \overline{O\,K}}{\overline{O\,L}} = \frac{A_r}{L_r\,l_r}$$

This is a quantity needed if the moments at the supports are directly graphically calculated by means of the "fixed points."

The length of the ordinate $\overline{W\,\omega}$ in both figures is independent of the intensity of the load. Hence if the load were of an elementary intensity $= p\,d\,x$ (where p is the weight per lineal unit and dx an elementary length) then the area formed by $\overline{W\,\omega}$ moving horizontally over the span would be proportional to A_r or B_r respectively for a uniform load over the portion moved over. It should be noticed however that the point ω moves in a curve that is tangent to the original equilibrium polygon at the point where the value of θ changes.

This completes the demonstration. It may be said to be

somewhat tedious but the same cannot be said of the graphical method. It is theoretically exact.

By it one may obtain results in a few hours that by the analytical or any other graphical method it might take weeks to accomplish and the author has found that the error need not exceed one or two per cent. if only ordinary care is used. The process throughout is as nearly mechanical as possible, thus not only preventing the danger of gross error but also getting rid of the burden of mental labor resulting from the algebraic numerical computations necessary in other methods.

SOME FACTS RELATING TO THE ARTICLE ON THE FORCE OF IMPACT OF WAVES AND THE STABILITY OF THE SUPERSTRUCTURE OF BREAKWATERS.

By L. d'Auria.

In the *Journal of the Franklin Institute*, for November, 1890, the writer, in his article "On the Force of Impact of Waves and the Stability of the Superstructure of Breakwaters," for the first time enunciated the following general principle: "The amount of energy which is necessary to cause the propagation of a wave is equal to the energy which such wave would require if it had to be moved *en masse* with its velocity of propagation upon a frictionless level surface" (p. 373.) This energy, when computed per linear foot of wave crest, is approximately expressed by

$$E = \frac{s\,k\,\gamma\,v^2}{4\,g}$$

in which s is the wave length; k its height measured from crest to trough; γ the weight of one cubic foot of water; and g the acceleration of gravity.

When a wave comes in contact with a breakwater the energy E is discharged against such structure in the interval of time which the wave requires to propagate its own length, so that, denoting by f the mean force of impact of

the wave against every foot of breakwater during such interval of time, we find

$$f = \frac{k \gamma v^2}{4 g}$$

According to observations the force of impact of waves against breakwaters is manifested only on that portion of the superstructure extending from crest to trough, so that, such force, at the time when the trough comes in contact with the breakwater, it must be zero and must gradually increase toward a maximum. Hence, the maximum force of impact per linear foot of superstructure can be expressed approximately by twice the mean, or by

$$F = \frac{\gamma k v^2}{2 g}$$

(See equation (1), *Journal Franklin Institute*, p. 374, November, 1890.)

The mean force of impact per square foot, deduced from this equation, is

$$p = \frac{\gamma v^2}{2 g}$$

and since the force of impact per square foot is found to vary from zero to a maximum, this maximum can be expressed approximately by twice the mean, or by

$$P = \frac{\gamma v^2}{g} \qquad (1)$$

This ought to accord with the maximum force of impact which can be observed by means of marine dynamometers placed against the superstructure of breakwaters. This has been proved to be a fact by the editor of *Engineering News*, in his editorial article on "Wave Pressure," published September 6, 1890, two months before the writer's article on the force of impact of waves, etc., appeared in the *Journal of the Franklin Institute*. This article, however, is a report the writer made to Col. Henry M. Robert, Corps of Engineers, U.S.A., while in the U. S. Engineer Office of Philadelphia, and was filed January 24, 1890. It was begun sometime in 1889 when the project to close the gap of the

Delaware Breakwater was being studied in that office. At that time the Assistant Engineer, Mr. L. Y. Schermerhorn, consulted the best works on the subject, but was unable to find any formula by means of which the maximum force of impact of a given wave against every linear foot of the superstructure of a breakwater could be ascertained, and the proper dimensions to be given to such superstructure determined. The want for such formulæ has always been felt by marine engineers, and it is no mean achievement for any one who succeeds in making such contribution to the dynamics of waves. The question now is how the editor of *Engineering News* reached the conclusion given in formula (1) of this paper, without the least knowledge of the energy of waves, as is shown by the manner in which he has handled the subject. Had there been no proposal for building the Diamond Shoal Lighthouse perhaps no one would have discovered that the editor of *Engineering News* possessed such valuable information regarding the wave pressure, as he calls it, for it was on this occasion, and immediately after the contract for that lighthouse had been awarded, that he presented himself as an authority in matters relating to wave pressure. This occurred on September 6, 1890, in his editorial already mentioned, in which, speaking of the bidders and the views they took in regard to wave action against the proposed lighthouse, he says: "There was evidently a decided contrast of views in regard to this question among the bidders on the structure, all of whom were engineers or contractors of high reputation."

Then, after reviewing the project of one of the bidders, he asks the question, "Are those allowances sufficient for a case of this kind?" To determine this he proceeds, saying, "Let us first run over certain general facts and theories," which he does, and accordingly reaches the conclusion, by Rankine's formula, that a wave 600 feet long, forty-three feet high, and moving at a rate of fifty-four feet per second, would probably at sea exert a pressure per square foot not exceeding 300 pounds. But he says: "The preceding laws hold only when the water is about as deep as the length of a wave or several hundred feet. Any person accustomed to surf-bath-

ing does not need to be told that the whole upper surface of the water moves forward with great speed on the crests and with hardly any in the troughs; while there is a constant undertow setting out from shore."

It seems that neither the bidders for the Diamond Shoal Lighthouse (all of whom are engineers or contractors of high reputation) nor any of the investigators or able marine engineers who have studied the subject in question, ever enjoyed the pleasure of a surf-bath, for otherwise they would have thought themselves warranted in assuming, as the editor of *Engineering News* assumes, without any other support, that "the surface of the sea in a harbor or on a bank moves forward with a velocity equal to that of the wave," and would have arrived at the "interesting results" found by him on such assumption. But let us see how he reaches these results. He quotes Rankine's *Civ. Eng.*, p. 754, in which this celebrated author says: "In water that is very shallow, compared with the length of the wave, the velocity is nearly independent of the length, and is nearly equal to that acquired by a heavy body in falling through half the depth of the water added to three-fourths of the height of a wave." The editor of *Engineering News*, however, instead of "very shallow compared with the length," substitutes, on his own authority, "when the depth of water is only a few times the height," and then proceeds to find the velocity of the waves observed at Wick, Scotland, which waves were forty feet high, and moved in water a trifle over sixty feet deep. Hence, he finds for such waves a velocity of sixty-two feet per second. Armed with this velocity, he considers the wave itself as a jet impinging upon the breakwater with such velocity, applies the formula for the impact of a jet of water against an unlimited plane surface, and comes out triumphantly with a wave pressure of 3·7 tons per square foot. Here he remembers for an instant that according to Rankine's formula, he had found that in the open sea the maximum pressure per square foot exerted by the greatest observed wave would not exceed 300 pounds, is staggered for a moment, and then remarks: "This result seems absurd as compared with that just deduced for the pres-

sure of waves in mid-ocean, but the absurdity is only apparent."

The reason given for this is that a maximum pressure observed by marine dynamometers is very nearly the one he has found by * * * experience in surf-bathing, in the absence of any theory. He, therefore, concludes: "Hence it will be seen that the assumption of a velocity of the particles of water equal to that of the wave, although not in accordance with theory, still leads to results agreeing with experiments." On the contrary, we have shown at the head of this paper that our theory leads exactly to the same results which are obtained by experiments. The principle upon which this theory is based has been proved to be correct in the article, "A New Theory of the Propagation of Waves in Liquids," which the writer has published in the *Journal of the Franklin Institute* for December, 1890. In this article, by the application of the same principle above-mentioned, a general formula for the velocity of propagation of any wave has been obtained, which, when applied to various cases, has shown the most remarkable agreement between practice and theory. Take, for example, the case of the tidal wave in which it is known that the horizontal displacement of the liquid particles from surface to bottom is constant, we have found that the velocity of propagation of such wave would be equal to that which a heavy body would acquire in falling through half the depth, a formula well known to those familiar with the subject, and which the editor of *Engineering News* evidently ignored. In fact, had he been acquainted with it, he would have understood Rankine differently and would have noticed that the generalization which this illustrious scientist had in view when he assumed the depth to be very shallow compared with the length of the wave, was referred to tidal waves, not to waves forty feet high and probably 500 feet long, moving in water sixty feet deep. In a wave of the class which was intended by Rankine and to which his generalization could be applied, the liquid particles are supposed to possess the same velocity of translation at the same instant and at all depths in the same cross-section, and the vertical motion must be supposed inconsider-

able in comparison with the horizontal motion. The tidal waves which are observed in tidal rivers are nearly waves of this kind, and the pressure which such waves exert upon a plane surface at right angle to their motion is almost insignificant compared with that due to their velocity of propagation.

It seems strange that the editor of *Engineering News*, while in a state of such ignorance about the whole subject of waves (with the exception of surf-bathing), should undertake to catechise "engineers and contractors of high reputation" on this very subject.

The most remarkable thing, however, about the incident, is that the editor of *Engineering News*, in the issue of November 15th, while engaged in the discussion of wave pressure with a correspondent, did not notice the writer's article on the force of impact of waves, etc., published in the *Journal of the Franklin Institute*, for November, 1890, though he noticed and reprinted from this *Journal* an article by Theo. N. Ely, on "Steam Heating of Passenger Trains," which ends on the same page where the article "On the Force of Impact of Waves," etc., begins. (See p. 373, *Journal of the Franklin Institute*, November, 1890.)

That the editor of *Engineering News* could fail to notice this article which, at that time, ought to have concerned him most, is impossible, and he did not even protest against the writer's arrogation of originality. Was it magnanimity, or was the editor of *Engineering News* aware of the fact that the writer's article had been filed in the United States Engineer Office of Philadelphia, January 24, 1890?

Indeed, this last alternative is not improbable, since the editor of *Engineering News* is not a stranger at that office. In fact, he is an old friend and schoolmate of the assistant engineer, with whom he often corresponds on engineering matters, and it may be that accidentally the conclusions reached by the writer in the above paper were propagated, so to speak, from Philadelphia to New York, which would solve the enigma without adding another word in the way of explanation.

PROCEEDINGS

OF THE

CHEMICAL SECTION,

OF THE

FRANKLIN INSTITUTE.

[*Stated meeting, held at the institute, Tuesday, December 16, 1890.*]

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, December 16, 1890.

Mr. T. C. Palmer, president, in the chair.

Members present: Prof. E. F. Smith, Prof. L. B. Hall, Dr. Wm. H. Wahl, Mr. Lee K. Frankel, Mr. H. Pemberton, Jr., Mr. Reuben Haines, Dr. S. C. Hooker, Prof. Henry Trimble, and several visitors.

Dr. Wahl, as chairman of the committee to prepare a circular to be addressed to the chemists of the country, reported progress, and the committee was continued. The secretary read his annual report to the section, which was accepted.

The election of officers followed, and, on motion, it was decided that the secretary cast the vote of the section for the ticket nominated at the November meeting. Dr. Wahl was escorted to the chair by the retiring president, and, on taking his seat, made a brief address appropriate to the occasion.

On motion of Dr. Hall, it was voted that the officers, president, secretary and treasurer, constitute three of the seven members of the committee on admissions.

The following gentlemen were then nominated and elected to fill the remaining vacancies in the committee, viz: Mr. H. Pemberton, Jr., Dr. S. E. Hooker, Prof. E. F. Smith, Dr. L. B. Hall.

The president appointed Messrs. Haines and Frankel to act with the secretary as members of the finance committee.

Dr. Wahl referred to the approaching convention of chemists, to be held shortly in this city, stating what had been done by the local committee to arrange for the event, and expressed the wish that the members of the section would second the efforts of the local committee to make the visitors welcome.

Dr. Greene exhibited specimens of the alloys of sodium and lead, discussed in the paper by himself and Dr. Wahl at the last meeting. They were examined with much interest.

Adjourned.

WM. C. DAY, *Secretary.*

On some Derivatives of Lapachic Acid (I).

By Samuel C. Hooker and Wm. H. Greene.

[*Read before the Chemical Section, March 18, 1890.*]

In a preliminary paper* published by the authors, it was shown that the conversion of lapachic acid into lapachone,† by the action of strong mineral acids, probably occurs in two stages, as indicated in the following equations:

$$C_{10}H_4\begin{cases}O_2\\CH = CH - C_3H_7\\OH\end{cases} + H_2O =$$

Lapachic Acid.

$$C_{10}H_4\begin{cases}O_2\\CH_2 - CH(OH) - C_3H_7\\OH\end{cases} =$$

Oxyhydrolapachic Acid.

$$C_{10}H_4\begin{cases}O_2\\CH_2 - CH - C_3H_7\\O\end{cases} + H_2O$$

Lapachone.

In support of this view it was stated that lapachone can be readily converted into oxyhydrolapachic acid, and that this acid, under the influence of mineral acids, again readily passes into its anhydride lapachone.

The details of these experiments, which have not yet been printed, will be published in this paper.

Conversion of Lapachone into Oxyhydrolapachic Acid.

The action of aqueous potash on lapachone has been studied by Paterno,‡ who writes as follows: "In aqueous potash of medium concentration lapachone does not dissolve in the cold; on heating it passes into a fine purple-red

* This *Journal*, **128**, 142.

† *Gazz. chim. Ital.*, **12**, 372.

‡ *Gazz. chim. Ital.*, **12**, 372.

solution which, filtered while hot, deposits on cooling beautiful orange needles of silky lustre, recognized by their fusing point (154°-155°) to be lapachone. On the addition of hydrochloric acid the alkaline filtrate yields a precipitate, which, purified by recrystallization, was similarly found to be lapachone."

This statement is misleading. The facts are these: Lapachone dissolves with some difficulty in hot aqueous potash, but in so doing undergoes a change. A new compound, an acid, is formed, and this, and not lapachone, exists in the solution obtained. If hydrochloric acid, in very slight excess, be added to the cold alkaline solution, the new compound separates as a yellow oil, which gradually assumes a crystalline form. If, however, a larger quantity of hydrochloric acid be employed, the color of the turbid solution is seen gradually to change. The bright yellow gives place to orange, and finally red crystals of lapachone may be observed floating in the liquid.

The new compound has, in fact, been reconverted, by the excess of hydrochloric acid employed, into lapachone. The experiment has been repeated a number of times, but the authors have not once observed the separation of crystals of lapachone from the alkaline solution as described by Paterno.

The action of potash on lapachone is shown in the following equation:

$$C_{10}H_4\begin{cases} O_2 \\ CH_2-CH.C_3H_7 \\ O \end{cases} + H_2O =$$

Lapachone.

$$C_{10}H_4\begin{cases} O_2 \\ CH_2.CH(OH).C_3H_7 \\ OH \end{cases}$$

Oxyhydrolapachic Acid.

In order to prepare oxyhydrolapachic acid, eight grams of lapachone, four grams of caustic potash and 150 cc. of water are heated together. If the crystals of lapachone be large, they should be first powdered. As the lapachone is

dissolved, the color of the solution becomes very intense, being similar to that of the salts of lapachic acid.

After boiling several minutes, the solution is filtered to remove any crystals which may have escaped the action of the potash, and an excess of acetic acid is then added. A yellow oil is immediately precipitated, which collects in the bottom of beaker, and then appears considerably darker than when first seen in a fine state of division. In the course of an hour or so, it solidifies to a yellow crystalline mass which, after some hours, may be separated and washed well with water. The acid, as thus obtained, though slightly colored at the surface, is in a very pure condition and can be rendered absolutely so by crystallization once or twice from alcohol, in which it dissolves very readily and from which it separates slowly in monosymmetric crystals. The yield is theoretical; eight grams of lapachone gave 8·4 grams of the crude acid; that required by theory being 8·59 grams.

The following figures were obtained on analysis:

(I) ·1965 gram gave ·4970 CO_2 and — H_2O
(II) ·4932 " " 1·2510 CO_2 " ·2743 H_2O
(III) ·2093 " " ·5306 CO_2 " ·1160 H_2O

	Found.			*Calculated for*
	I.	*II.*	*III.*	$C_{15}H_{16}O_4$.
C, ...	68·96	69·18	69·13	69·23
H, ...	lost	6·18	6·15	6·15

Oxyhydrolapachic acid melts at 125°. It is readily soluble in most of the ordinary solvents, from which it crystallizes after standing some time; if, however, the solvent be allowed to evaporate rapidly, the acid is left as a yellow oil. Under ordinary conditions it is a perfectly stable body, which can be crystallized from acetic acid unchanged; but in contact with mineral acids, even when dilute, it is readily converted into its anhydride lapachone, which was identified by its fusing point and other properties, and by analysis.

The barium salt is extremely characteristic, separating from a claret-colored solution in bright *orange*, silk-like needles, grouped together in wavy tufts. In order to prepare it, the acid was dissolved in a solution of baric hydrate,

from which the excess of barium was precipitated at the boiling temperature by means of carbon dioxide. The solution was then concentrated to the crystallizing point. As the evaporation proceeds, a film of the salt, in an amorphous condition, and of the *same* color as the solution, forms at the edge of the liquid, on the evaporating basin. The salt is much more soluble in hot than in cold water. After recrystallization, the orange needles gave the following figures on analysis:

(I) ·2152 gram substance gave ·0744 $BaSO_4$.
(II) ·2883 " " " ·0987 "
(III) ·2478 " " " ·0854 "

	Found.			*Calculated for*	
	I.	*II.*	*III.*	$(C_{15}H_{15}O_4)_2Ba, H_2O.$	$(C_{15}H_{15}O_4)_2Ba.$
Ba (per cent.), .	20·32	20·12	20·26	20·35	20·91

The above figures were obtained from different preparations, dried over sulphuric acid, and also at 110°.

The salt evidently contains one molecule of water of crystallization, although the attempts to determine this by loss were unsuccessful. Heated to 110° for several hours, no change in weight occurred, and decomposition commenced at somewhat higher temperatures. It was, however, observed that the orange crystals, when rubbed, became dark magenta-red, and analysis proved the altered salt, dried at 105°–110°, to be anhydrous. Under the microscope, it showed no definite structure, and when moistened with water it became immediately orange, at the same time swelling up and then passing into solution. On evaporation, the characteristic orange needles of the salt, as above described, were again obtained.

The anhydrous salt was prepared for analysis by thoroughly grinding the orange crystals, in small quantities at a time, in an agate mortar. The conversion was not easily accomplished, and it seemed as if a resinous substance were being dealt with. The ground substance was heated to 110° until constant in weight.

(I) ·3368 gram substance gave ·1182 $BaSO_4$.
(II) ·2834 " " " ·1002 "

	Found.		*Calculated for*
	I.	*II.*	$(C_{15}H_{15}O_4)_2Ba.$
Ba, ...	20·63 p.c.	20·78 p.c.	20·91 p.c.

The two determinations were made with different preparations.

The calcium salt is entirely different from the barium salt just described. It separates from its solution on envaporation in small dark-red crystals, and is only very sparingly soluble in both hot and cold water. After drying over sulphuric acid, it did not lose in weight when exposed for over an hour to a temperature of 105°–110°. Analysis shows it to be anhydrous.

(I) ·3113 gram gave ·0808 $CaSO_4$
(II) ·2594 " " ·0635 "

	Found. I.	II.	*Calculated for* $(C_{15}H_{15}O_4)_2Ca$.
Ca., ...	7·63 p.c.	7·19 p.c.	7·16 p.c.

The preparations analyzed were different. (I) was obtained by adding the theoretical quantity of calcium chloride to the slightly ammoniacal solution of the acid; on expelling the excess of ammonia by heat, the salt separated. (II) was prepared as above described for orange variety of the barium salt.

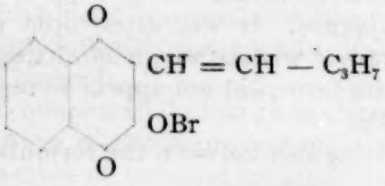
The silver salt was obtained by precipitating a concentrated neutral solution of the ammonium salt with the calculated quantity of silver nitrate. It separates in a semi-resinous condition, becoming granular after some time. It is decidedly soluble in water, also in dilute alcohol, from which it separates, on spontaneous evaporation, in small dark-red needles. The figures it gave on analysis lie between those required for $C_{15}H_{15}O_4Ag$ and $C_{15}H_{15}O_4Ag + H_2O$. It is probable, therefore, that the compound was not obtained in a pure condition.

ON SOME DERIVATIVES OF LAPACHIC ACID (II).

BY SAMUEL C. HOOKER.

[*Read before the Chemical Section, Franklin Institute, May 20, 1890.*]

Through the action of bromine on lapachic acid in acetic acid solution, Paterno* obtained a compound $C_{15}H_{13}BrO_3$, which he regarded as bromlapachic acid, and in which he believed the bromine to have replaced the hydroxylic hydrogen of lapachic acid. He, therefore, assigned to it the formula†



In selecting this formula, Paterno was guided by the following reactions:

(1) Lapachic acid dissolves readily in alkalies in the cold, but bromlapachic acid does not.

(2) The monacetyl derivative of lapachic acid yields on treatment with bromine bromlapachic acid.

(3) On oxidation with nitric acid phthalic acid, not bromphthalic acid, is formed.

(4) Unlike lapachic acid, bromlapachic acid undergoes no changes in contact with concentrated sulphuric acid.

In view of the light thrown on the formation of lapachone from lapachic acid by the author in conjunction with Dr. Wm. H. Greene,‡ it seems probable that the action of bromine on lapachic acid takes place as indicated in the

* *Gazz. chim. Ital.*, **12**, 353.

† It still remains an open question whether lapachic acid is derived from α- or β-napthaquinone. Paterno is occupied in solving this question. See also preceding paper.

‡ This *Journal*, **128**, 147.

following equations, and that the resulting compound is bromlapachone, and not bromlapachic acid.

$$C_{10}H_4 \begin{cases} O_2 \\ CH = CH - C_3H_7 \\ OH \end{cases} + Br_2 =$$

$$C_{10}H_4 \begin{cases} O_2 \\ CHBr - CHBr - C_3H_7 \\ OH \end{cases} =$$

$$C_{10}H_4 \begin{cases} O_2 \\ CHBr - CH \cdot C_3H_7 \\ O \end{cases} + HBr$$

This view was advanced by Hooker and Greene* about a year ago, without, however, being supported by direct experimental evidence. It was afterwards discussed by Paterno and Minunni† who, although admitting it had some strong points in its favor, did not appear to regard it on the whole as probable.

In order to distinguish between the formulæ

$$C_{10}H_4 \begin{cases} O_2 \\ CH = CH - C_3H_7 \\ OBr \end{cases} \quad (1)$$

and

$$C_{10}H_4 \begin{cases} O_2 \\ CHBr - CH - C_3H_7 \\ O \end{cases} \quad (2)$$

the action of potash on the so-called bromlapachic acid has been studied, as it seemed probable that, if its constitution is correctly represented by the formula (1), lapachic acid would be again formed by the removal of the bromine. If, on the other hand, the compound is a derivative of lapachone it should behave similarly to lapachone (see preceding paper) in contact with potash.

The results obtained prove unmistakably that Paterno's so-called monobromlapachic acid is in reality monobromlapachone.

The action of dilute potash upon the compound at

* This *Journal*, **128**, 147.

† *Gazz. chim. Ital.*, **19**, 623.

the boiling temperature is represented in the following equation:

$$C_{10}H_4 \begin{cases} O_2 \\ CHBr - CH - C_3H_7 \\ O \end{cases} + 2H_2O =$$

Bromlapachone.

$$C_{10}H_4 \begin{cases} O_2 \\ CH(OH) . CH(OH) . C_3H_7 \\ OH \end{cases} + HBr$$

Dioxyhydrolapachic Acid.

If bromlapachone had the formula which Paterno assigns to it as bromlapachic acid, it should form an addition compound with bromine. The author has allowed it to remain for several days in contact with the theoretical quantity of bromine in chloroform solution, but has not observed the formation of any addition compound. Bromlapachone was recovered unchanged.

In order to remove all doubts as to the mechanism of the reaction resulting in the formation of bromlapachone, the study of the action of bromine on lapachic acid in chloroform solution has been undertaken in the hope of isolating the intermediate product

$$C_{10}H_4 \begin{cases} O_2 \\ CHBr - CHBr . C_3H_7 \\ OH \end{cases}$$

In the course of this investigation no less than five compounds have been obtained by the action of one molecule of bromine on one molecule of lapachic acid. The study of these interesting bodies has had to be temporarily suspended, owing to dearth of material, but it is hoped that the work will be soon resumed and the results soon ready for publication.

By the action of dilute mineral acids upon dioxyhydrolapachic acid it is converted with great ease into its anhydride, oxylapachone, as follows:

$$C_{10}H_4 \begin{cases} O_2 \\ CH(OH) . CH(OH) . C_3H_7 \\ OH \end{cases} =$$

$$C_{10}H_4 \begin{cases} O_2 \\ CH(OH) . CH . C_3H_7 \\ O \end{cases} + H_2O$$

Further, just as oxyhydrolapachic acid results from the action of alkalies on lapachone, so dioxyhydrolapachic acid results from their action on oxylapachone.

BROMLAPACHONE.

In the course of some experiments, already referred to, it was observed that, under certain conditions, bromlapachone (Paterno's monobromlapachic acid) could be readily obtained by the action of bromine on lapachic acid in chloroform solution. As the yield is much larger and the substance purer than when prepared in acetic acid solution, as described by Paterno, this method of preparation was employed. It will be described in detail when the compounds arising from the action of bromine on lapachic acid in chloroform solution are discussed.

The bromlapachone was first carefully compared with that prepared as directed by Paterno, and in all particulars was found to be identical. A bromine estimation was made, with the following result:

·2588 grm. substance gave ·1502 AgBr.

	Found.	*Calculated for* $C_{15}H_{13}BrO_3$.
Br, ...	24·69	24·92

Bromlapachone has been observed by Paterno to separate from its solution in alcohol in orange-red crystalline plates. While able to confirm this observation, the author has also obtained it, when in a pure form, crystallized in tufts of fine needles. The conditions under which both varieties of crystals are formed may be stated generally as follows:

From slightly impure alcoholic solutions bromlapachone separates in plates. The same is true whether the solution be allowed to remain at perfect rest, or be disturbed while crystallization is proceeding.

From pure concentrated solutions bromlapachone can be readily obtained in plates by keeping the solution slightly in motion while the compound is crystallizing out.

If the compound, crystallized in plates and still in contact with its saturated mother liquor, be allowed to stand one or more weeks, the plates will be gradually replaced by tufts of needles. This change is believed to occur only when the

compound is in a very pure condition. The observation was first accidentally made, and the experiment was then repeated several times.

Pure solutions protected from dust and allowed to stand absolutely at rest frequently deposit the compound crystallized entirely in tufts of needles.

By observing only partially the above conditions both varieties of crystals may often be obtained in some solution.

The above remarks apply to all solutions of bromlapachone, whether obtained from needles or plates.

The melting points of the two varieties of crystals differ only very slightly, if at all. That of the plates was found to be 138°·5; that of the needles, 138°.

A combustion of the needles gave the following results:

·2399 grm. substance gave ·4932 CO_2 and ·0870 H_2O.

	Found.	*Calculated for* $C_{15}H_{13}BrO_5$
C,	56·06	56·07
H,	4·02	4·04

CONVERSION OF BROMLAPACHONE INTO DIOXYHYDROLAPACHIC ACID.

As the result of several experiments 1·5 grams of finely-powdered bromlapachone and 75 cc. of a one per cent. solution of caustic potash were boiled together with an inverted condenser. The bromlapachone dissolved slowly to an intense carmine-red solution. At the end of fifteen minutes the boiling was discontinued and the solution filtered. Stray crystals of bromlapachone, which had escaped grinding and were consequently able to resist the action of the potash for a long time, were thus removed. It was observed that a small quantity of a green substance, formed by the action of the potash, was also retained on the filter paper. A slight excess of acetic acid was added to the filtered solution when cold, resulting in the immediate formation of a precipitate in small quantity, consisting of a somewhat resinous brown substance. This was removed as quickly as possible by filtration through a folded paper of comparatively large size. The filtration must be promptly accomplished, otherwise the new acid will commence to crystallize out before

the separation of the brown impurity has been effected. In a very minutes after acidifying the acid commences to be deposited from the now orange-red solution in compact crystalline grains. The separation occurs slowly, the solution becoming simultaneously lighter in color. In the course of a few hours the crystallization is complete and the compound can be filtered off, washed well with water and dried. The acid, as thus obtained, is almost pure: 1·5 grams bromlapachone gave rather more than one gram of the new acid. It was twice recrystallized from ninety-five per cent. alcohol, in which it is not very soluble, and analyzed. A qualitative test showed that in spite of the diluteness of the potash solution used the bromine had been removed.

·1867 gram gave ·4442 CO_2 + ·0989 H_2O.

	Found.	*Calculated for* $C_{15}H_{10}O_5$.
C,	64·88	65·21
H,	5·88	5·79

Dioxyhydrolapachic acid melts at 181°–182°. Like bromlapachone, it crystallizes from ninety-five per cent. alcohol in two apparently different forms. No special experiments were made to determine the conditions governing the formation of each variety of crystal and sometimes the one form, sometimes the other, would be obtained under apparently similar conditions. One modification (that analyzed) consists of small prismatic crystals, often grouped together in the form of compact stars; the other, of very fine long needles, crystallizing in tufts.

This acid, like oxyhydrolapachic and lapachic acids, forms stable salts which are not decomposed by carbon dioxide. The color of their solutions is similar to that of those of lapachic acid. The small amount of available material did not permit a more complete study of these salts. Dioxyhydrolapachic acid dissolves somewhat in water and is soluble in all ordinary solvents.

CONVERSION OF DIOXYHYDROLAPACHIC ACID INTO OXYLAPACHONE.

While dioxyhydrolapachic acid is perfectly stable under ordinary conditions and shows no inclination whatever to

pass into its anhydride, it forms oxylapachone with great readiness in contact with mineral acids.

To a solution of 1·36 grams of the acid * in the smallest possible quantity of boiling 95 per cent. alcohol, 5 cc. of concentrated hydrochloric acid were added: the color of the solution changed to an intense orange-red. After boiling for one or two minutes the solution was diluted with water as long as a precipitate was produced. A considerable portion of the oxylapachone was thus precipitated in small orange-red needles. The filtrate, still orange in color, was evaporated until a crust of crystals had formed on its surface. They were separated from the solution when cool. Total yield, 1·15 grams. The substance was purified for analysis by recrystallization from dilute alcohol; it separated in red needles, fusing at 201°·5, and closely resembling lapachone in appearance.

·2507 gram gave ·6401 CO_2 + ·1228 H_2O.

	Found.	*Calculated for* $C_{15}H_{14}O_4$.
C,	69·63	69·76
H,	5·44	5·42

CONVERSION OF OXYLAPACHONE INTO DIOXYHYDROLAPACHIC ACID.

Oxylapachone, like lapachone and bromlapachone, is almost insoluble in alkalies in the cold; on boiling, however, it readily passes into solution. Thirty-five cc. of a one per cent. solution of caustic potash, and 1·35 grams oxylapachone, were heated for several minutes at the boiling point. The solution was then filtered, cooled and acidified with acetic acid. Crystals commenced to form almost immediately, but the separation was not complete for some hours. After crystallization from alcohol the compound fused at 181°–182°, and in other respects was found to be identical with dioxyhydrolapachic acid, obtained as above-described from bromlapachone. The yield of the crude substance was

* The whole of the substance available after a few preliminary experiments had been made on a still smaller scale.

almost theoretical; 1·35 grams gave 1·36 grams of the acid, 1·45 being required by theory.

·1834 gram gave ·4390 CO_2 + ·0958 H_2O.

	Found.	*Calculated for $C_{10}H_{10}O_3$.*
C,	65·28	65·21
H,	5·80	5·79

The following is a list of the acids and their anhydrides discussed in this and the preceding paper:

Melting Point.	*Acid.*	*Anhydride.*	*Melting Point.*
139°	$C_{10}H_4\begin{cases}O_2\\CH=CH-C_3H_7\\OH\end{cases}$ Lapachic Acid.		
125°	$C_{10}H_4\begin{cases}O_2\\CH_2-CH(OH)-C_3H_7\\OH\end{cases}$ Oxyhydrolapachic Acid.	$C_{10}H_4\begin{cases}O_2\\CH_2-CH.C_3H_7\\O\text{———}\end{cases}$ Lapachone.	155–6°
181–2°	$C_{10}H_4\begin{cases}O_2\\CH(OH).CH(OH).C_3H_7\\OH\end{cases}$ Dioxyhydrolapachic Acid.	$C_{10}H_4\begin{cases}O_2\\CH(OH).CH.C_3H_7\\O\text{———}\end{cases}$ Oxylapachone.	201°
	Unknown.	$C_{10}H_4\begin{cases}O_2\\CHBr-CH.C_3H_7\\O\text{———}\end{cases}$ Bromlapachone.	138°

The acids are bright yellow in color, the anhydrides are orange or red. This distinction in color extends to other lapachic acid and lapachone derivatives which the author has obtained, but which have not yet been sufficiently studied to justify their introduction here.

NOTE ON THE COMPOUND OF PYRROL WITH PICRIC ACID.

By SAMUEL C. HOOKER.

[*Read before the Chemical Section, February 18, 1890*].

In a short paper, devoted to some similar reactions of carbazol and pyrrol,* the author described, nearly two years ago, a compound which he had obtained from pyrrol and picric acid, fusing at about 71° and crystallizing in long red needles. The method of formation, as well as the properties of this compound, justified the assumption that it was an addition-product similar to the picric acid compounds of indol and carbazol.

Owing to the great instability of the compound, its analysis promised considerable difficulty, and was consequently deferred until the required leisure would make it possible to devote to it the necessary time and care.

By following the directions here given, no difficulty will be found in preparing the compound in a pure form: Freshly-distilled pyrrol is added to a small quantity of picric acid, in quantity a little more than may be necessary to cover it. As soon as the pyrrol comes in contact with the acid, the latter becomes intensely red. The temperature, which must at no time exceed 58°, is then gradually raised until the picric acid is completely dissolved. The heating must not be continued longer than is actually necessary for the picric acid to pass into solution. A short time after the temperature has fallen the compound separates in red needles, some long, others short and broad.

If a sufficient quantity of pyrrol be used in proportion to the picric acid and care be taken to heat as short a time as possible, the crystals are very easily freed from the mother liquor by pressure between filter paper. If, on the other hand, these points are not attended to, the mother liquor is

* This *Journal*, **127**, 63.

so thick and sticky that simple pressure does not suffice to leave the crystals in a pure condition.

To prepare the substance for analysis the mother liquor was drained off and the crystals were then rapidly and repeatedly pressed between porous paper until the paper was scarcely, or not at all, stained. The substance was then placed between well-fitting watch-glasses and immediately weighed. As soon as the crystals are dry, they commence to decompose, giving off pyrrol; they were, therefore, simply exposed to the air after the first weighing and then weighed again until the weight remained constant. The loss in weight represented the pyrrol given off, the residue consisted of picric acid.

(I) ·3230 gram lost ·0740 gram.
(II) ·5350 " " ·1210 "
(III) ·4460 " " ·1015 "

	Found.			*Calculated for*
	I.	*II.*	*III.*	$C_4H_5N, C_4H_3N_3O_7$
Pyrrol,	22·91	22·61	22·75	22·63
Picric acid,	77·09	77·39	77·25	77·37

That pyrrol was really given off from the crystal was ascertained by holding a pine splinter, moistened with hydrochloric acid, in the neighborhood of crystals which had been exposed for two or three hours (presumed to have been sufficiently long to have allowed all possible traces of pyrrol simply adhering to the crystals to have escaped); the characteristic reaction was obtained.

The residue was of a very pale yellow color, and fused, at the same time darkening, at about 120°. It dissolved almost entirely in water, leaving a minute quantity of a dark resin. Crystallized once from water it fused* at 120°–121°. The properties of the residue—its fusing point, the intense color and intensely bitter taste of its solution, its crystalline habits, etc.—corresponded exactly with those of picric acid.

Alcoholic solutions of pyrrol and picric acid deposit red crystals of the addition compound on spontaneous evaporation. The same is true of a chloroform solution. The combination does not appear to take place when benzol

* The fusing point of picric acid is 122°·5.

is the solvent employed.* Considering the very unstable nature of the pyrrol compound, it is not difficult to understand how the slight affinity, which exists between pyrrol and picric acid, may be overcome in the presence of relatively large quantities of benzol which itself forms an unstable compound with picric acid.

ON SOME CONSTANTS OF AMMONIA.†

BY DR. HANS VON STROMBECK.

[*Read at the stated meeting of the Chemical Section, October 21, 1890.*]

[*Concluded from vol. c, p. 481.*]

III.—DETERMINATION OF THE HEAT OF ABSORPTION OF AMMONIA.

The tests were made in the following way: The gas is developed by liquid ammonia contained in tank *AB* (*Fig. 3*). In order to have an as constant development of gas as possible during the test, tank *AB* is put in tank *CD* filled with water. Cock *Z* is of the same construction as cock *Z* in the preceding tests. The gas has to pass through the low stratum of mercury in the glass tube *EF*, and so its velocity can be controlled exactly. From hence the ammonia gas has to pass through the glass-coils *GH*. By passing through these coils lying in tank *IK*, the water of which is always renewed, the gas enters the absorption tank *NOPQ* at an almost constant temperature. The pipe *RS* reaches almost to the bottom of the tank and dips into a small glass tube secured in a piece of cork and filled with mercury, in order to prevent the water from rushing up in this pipe after it came in contact with the ammonia.

In each test 11,000 grammes of water are put in the absorption tank; its value in water being 245 grs., the weight of water (*M*) which is to be heated up is 11,245 grs. The quantity of ammonia gas (*P*) absorbed by the water is obtained by taking at the end of the test 100^{ccm} from the

* M. Dennstedt, *Ber. d. chem. Ges.*, **21**, 34, 31.

† From the laboratory of the De La Vergne Refrigerating Machine Co.

absorption tank, determining its contents of ammonia and multiplying the figure by 110.

I shall add to each item the figures obtained in the second test.

If in the following:

P is the weight of ammonia which was absorbed (108·67 grs.);

θ is the temperature of the water in the absorption tank at the commencement of the test (15°·88);

$\theta_1 + \delta$ is the corrected temperature of the water in the absorption tank at the end of the test (20°·79);

$\frac{\theta + \theta_1}{2}$ is the average temperature prevailing in the absorption tank during the test (17°·84);

$\frac{\tau + \tau_1}{2}$ is the average temperature of the ammonia on entering the absorption tank (17°·31);

$\frac{\theta + \theta_1}{2} - \frac{\tau + \tau_1}{2}$ is the difference of both temperatures (0°·53);

$\theta - (\theta_1 + \delta)$ is the number of degrees by which M was heated up (4°·88);

c is the specific heat of gaseous ammonia (0·5084).

The heat (Q) developed by P is

$$M[\theta - (\theta_1 + \delta)] = 54875{\cdot}6 \text{ thermal units.}$$

But we have to make one correction, it is to be added:

The quantity of heat (s) required by P for heating itself up from

$$\frac{\tau + \tau_1}{2} \text{ to } \frac{\theta + \theta_1}{2};\ s = Pc\left(\frac{\theta + \theta_1}{2} - \frac{\tau + \tau_1}{2}\right) = 29{\cdot}3$$

Consequently we have

$$Q = 54875{\cdot}6 + 29{\cdot}3 = 54904{\cdot}9$$

or the heat developed by the unit (μ) is equal to

$$\frac{Q}{P} = 505{\cdot}3 \text{ thermal units.}$$

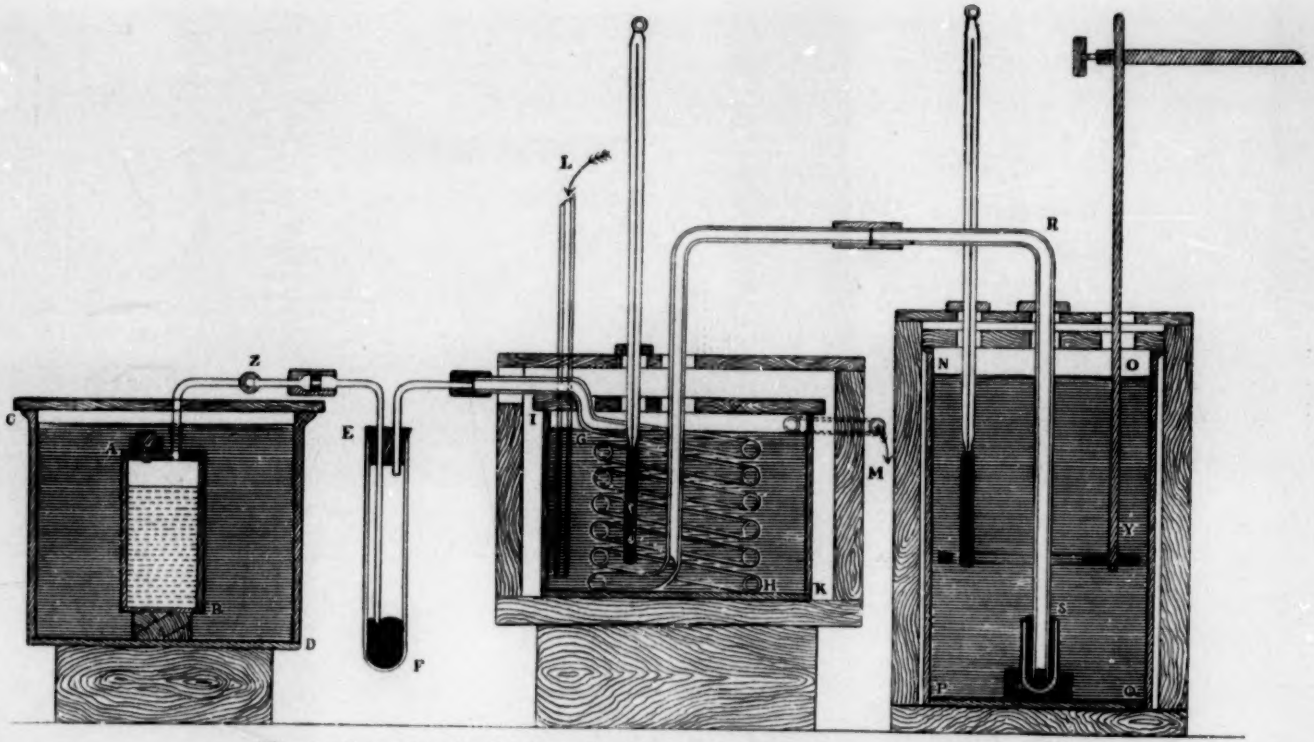


Fig. 3.—Apparatus for the determination of the heat of absorption of ammonia.

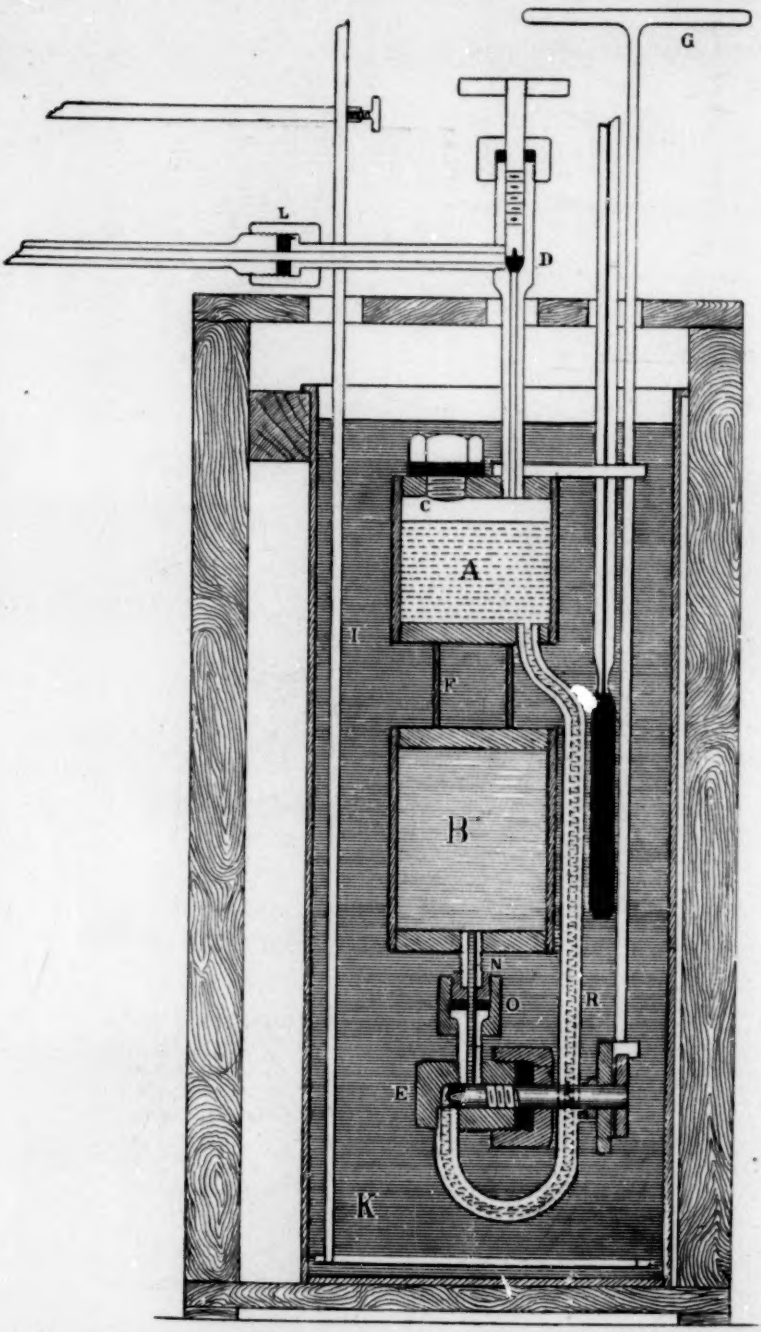


Fig. 4.—Apparatus for the determination of the heat of combination of ammonia.

AUTHENTICAL PROOFS OF THE DETERMINATION OF THE HEAT OF ABSORPTION OF AMMONIA.

	1.	2.	3.	4.	5.	6.	7.	8.	AVERAGE.
P, ...	112·25	108·67	110·1	148·5	134·0	100·54	152·2	170·5	
M, ...	11245·0	11245·0	11245·0	11245·0	11245·0	11245·0	11245·0	11245·0	
θ, ...	13·89	15·88	15·33	12·88	13·83	15·91	12·66	12·58	
$\theta_1 + \delta$, ...	19·20	20·79	20·31	19·68	20·06	20·61	19·80	20·65	
$\frac{\theta + \theta_1}{2}$	15·52	17·84	16·75	15·55	17·01	18·09	15·91	16·23	16°·61
$\frac{\tau + \tau_1}{2}$	14·693	17·31	15·59	14·96	15·85	16·06	15·27	16·00	
$\frac{\theta - \theta_1}{2} - \frac{\tau + \tau_1}{2}$	0·827	0·53	1·16	0·59	1·26	2·03	0·64	0·23	
$\theta - (\theta_1 + \delta)$, ...	5·175	4·88	4·7009	6·6315	5·944	4·5281	6·6704	7·4446	
$M(\theta - (\theta_1 + \delta))$,	58192·9	54875·6	55110·6	74571·2	66840·3	50918·5	75008·6	83714·5	
s, ...	47·3	29·3	66·3	46·1	87·7	104·1	47·7	18·6	
Q, ...	58241·4	54904·9	55177·9	74618·7	66928·6	51023·6	75057·7	83734·4	
$\mu = \frac{Q}{T}$	518·8	505·3	501·1	502·3	499·1	507·7	493·1	491·1	{ 502·3 French or 904·1 Engl. thermal units.

IV. DETERMINATION OF THE HEAT OF COMBINATION OF LIQUID AMMONIA AND WATER.

The tests were made in the following way: To tank *A* (*Fig. 4*) which through the hole *c* is to be filled with ammonia *P* is secured the pipe *R*, which has at its end the cock *E* and the union *O*; cock *E* can be handled by *G*. Union *O* can be connected with union *N*, terminating into the bottom of tank *B*, which is to be filled with water. Support *F* serves the purpose of making the whole apparatus steadier. I first made the connection between the upper ammonia- and the lower water-tank by a straight pipe going from the bottom of the upper to the top of the lower tank. In its middle a cock was placed which could be handled in the same way as the now present cock. But in this case at the place of contact between both fluids, a saturated stratum of aqueous ammonia is formed which, being heavier than liquid ammonia and lighter than water, does not move at all, but stays where it is, thus preventing both fluids from combining with one another. On the other hand, it will not do to put the liquid ammonia in the lower tank and the water in the upper one, for, it being next to impossible to fill a tank entirely with a fluid of such a low boiling point as liquid ammonia, there would always be left a small space filled with gaseous ammonia which, of course, would be at the top of the lower tank. Consequently, we would not determine the heat of combination but would again determine the heat of absorption.

In the following I shall add to each item the figures obtained in the first of the five tests I made.

Tank *A* is weighed empty, cocks *E* and *D* being closed, and *L* shut by a plug. Then it is filled with as much liquid ammonia as possible and weighed again. The difference between both weights is the one of ammonia *P* (53·7 grs.). At the end of each test it remains filled with ammonia gas ($P—P_1$) at atmospheric pressure. The contents of *A* being 160^{ccm}, $P—P_1$ weighs $160 \times 0{\cdot}0007614 = 0{\cdot}1218$ grs., which are to be subtracted from *P*, so as to obtain P_1 (53·58 grs.). After *A* is filled it and tank *B* are put in a large vessel with water, and after, by proper manipulations, *B* as well as the

connecting pipe between A and B are entirely filled with water, both are connected. The whole apparatus being put in tank K filled with a certain quantity of distilled water, L is connected with a gauge after the plug is removed. The agitator I is put in motion, and the thermometer and the gauge are observed. As soon as for about five minutes the temperature and the pressure have become almost constant, cock E is opened. The pressure goes rapidly down to the atmosphere, and the thermometer ascends quickly, the temperature indicated by it soon becoming almost constant. From the figures thus obtained the heat of combining (ϕ) can be calculated.

The quantity of water (M) which is to be heated up in each test, consists of:

		grs.
The value in water of the apparatus, . . .		567·4
The weight of the water in tank B, . . .		237·0
The weight of the water in tank K, . . .		8200·0
The weight of the liquid ammonium in tank A, multiplied by its specific heat, . . .	$53{\cdot}58 \times 1{\cdot}22876 =$	65·8
	$M =$	9070·2

If

θ = the temperature of the water in tank K at the commencement of the test (20°·60);

θ_1 = the temperature of the water in tank K at the end of the test (21°·77);

$\theta - \theta_1$ = the difference between both temperatures (1·170);

p^{θ} = the absolute pressure in atmospheres prevailing at the commencement of the test (8·69);

p^{θ_1} = the absolute pressure in atmospheres prevailing at the end of the test (1·0);

the heat developed by the combining of P_1 (53·58 grs.) with water is equal to $M(\theta - \theta_1) = 10612{\cdot}1$ or the heat (ϕ) developed by the unit is

$$\phi = \frac{M(\theta - \theta_1)}{P_1} = 198{\cdot}1 \text{ thermal units.}$$

I did not take into consideration the difference of weight

between the few cubic centimeters of gaseous ammonia being over the liquid at the commencment of the test at p^{θ} and the 160^{ccm} of gaseous ammonia filling tank *A* at the end of the test at p^{θ_1}, because its influence on the result, if any, can only be minimal.

According to the equivalents of NH_3, H_2O, 17 grs. of the former combine with 18 grs. of the latter to 35 grs. of $NH_4(OH)$. Therefore, from a theoretical point of view, it would be best to use ammonia and water in these proportions in a test. But, because of technical reasons, the surface of contact between ammonia and water in an apparatus can be only small, so that the diffusion between both will take place rather quickly only then if the specific gravity of the resulting aqueous ammonia is a great deal greater than the one of liquid ammonia; for in this case there is always in *B* a descending current of aqueous ammonia and an ascending current of liquid ammonia, and fresh water comes always into contact with fresh ammonia. I, therefore, took the dimensions of tanks *A* and *B* in such a way that the ratio between the combining ammonia and water was, instead of 17 : 18, only about 8 : 18.

The results obtained in the other four tests were:

	2.	3.	4.	5.	Average of all 5 tests.
P,	59·2 grs.	60·22	55·12	60·20	
P_1,	59·08 grs.	60·10	55·00	60·08	
$P - P_1$,	0·12 grs.	0·12	0·12	0·12	
M,	9276·5 grs.	9277·7	9271·1	9277·7	
θ,	21°·29	23°·94	19°·62	21°·16	
θ_1,	22°·48	25°·29	20°·84	22°·47	
$\theta - \theta_1$,	1°·19	1°·35	1°·22	1°·31	
$M(\theta - \theta_1)$,	11039·0	12524·9	11310·7	12153·9	
p^{θ},	8·75 atm.	9·68	8·39	8·52	
p^{θ_1},	1	1	1	1	
ϕ,	186·9	208·4	205·6	202·3	200·3 French or 360·5 Eng. thermal units.

V. TABLE, *Showing the amount in French and English thermal units required or developed by liquid or gaseous ammonia when changing their respective states of aggregate and for what these thermal units are used.*

Total heat required by liquid ammonia to transform itself at 17°·0 (62°·60 F.) into saturated vapor, its boiling point being in reality at — 0°·86 (30°·45 F.), ...	— 318·8 (573·8)	— 296·8 (534·2) heat of evaporation. — 22·0 (39·6) heat of liquid.
Heat developed by the absorption of gaseous ammonia by water at atmospheric pressure, the temperature of the resulting aqueous ammonia being 16°·6 (61°·9 F.), .	+ 502·3 (904·1)	+ 318·8 (573·8) to become a liquid at — 0°·86. + 46·2 (83·2) to cool itself down from — 0·86° to its boiling point at atmospheric pressure to — 38°·5 = 37·64 × 1·22876. — 67·6 (121·7) to heat itself up from — 38°·5 to 16°·6 = 55·1 × 1·22876. + 200·3 (360·5) for chemical work done by the combination of liquid ammonia and water.
		+ 497·7 (895·8)

BOOK NOTICES.

Handy lists of technical literature.—Reference catalogue of books printed in English from 1880 to 1888, inclusive, to which is added a select list of books printed before 1880 and still kept on publishers' and jobbers' lists. Part II[a]. Electricity and magnetism, telegraph, gas, etc. Compiled by H. E. Haferkorn, Milwaukee. 1890.

This, the fourth part of the valuable set of handy lists, is arranged on the same basis as former ones, including a large number of cross-references, which aid greatly in finding any particular work; there are also notes and remarks, giving such information as is necessary to complete each entry. The entire catalogue is supplemented by a list of newspapers, periodicals, almanacs and annuals.

Being the first attempt made at cataloguing the later literature of this extensive subject, we are surprised to find it so complete.

It will recommend itself to all who are interested in electricity, magnetism or gas. R.

Sugar analysis. For refineries, sugar houses, experimental stations, etc., and as a hand-book of instruction in schools of chemical technology. By Ferdinand G. Wiechmann, Ph.D. New York: John Wiley & Sons. 1890. Price, $2.50.

This book forms a valuable supplement to the works on sugar analysis already in use. Some important matters, however, have been insufficiently

discussed, while others have been entirely overlooked. One brief page is devoted to the methods of examining how effectively sugar has been decolorized in passing through bone-black; but a description of the process of decolorizing sugar for comparative tests, which should precede this, has not been given.

Bone-black itself has, it is much to be regretted, been almost entirely passed over. On p. 107, we are shown how the weight of a cubic foot of this substance may be ascertained; but with this exception there is little or nothing said about it. It is true that the analysis of bone-black is foreign to the subject-title, but in view of the almost inseparable connection of sugar and bone-black, the value of the work would have been much increased, both to the practising chemist and also as a "hand-book of instruction in schools of chemical technology," if some space had been devoted to it. We find no mention of either the sugar-cane or the sugar-beet; but this we regard as a very important omission. A chapter might advantageously have been devoted to the detection of foreign substances in sugar. Thus, a description of the best methods of finding tin, ultramarine, the various artifical coloring matters, etc., would have added to the value of the book.

After recording these and passing over similar omissions, we can highly commend the book to all interested in sugar analysis, as bringing down the subject to the present date in a concise and practical form. The book contains a valuable and very complete set of tables. S. C. H.

Franklin Institute

[*Proceedings of the stated meeting, held Wednesday, December 17, 1890.*]

HALL OF THE FRANKLIN INSTITUTE,
PHILADELPHIA, December 17, 1890.

JOS. M. WILSON, president, in the chair.

Present, 192 members and forty-three visitors.

Additions to membership since last report, forty.

The actuary, by direction of the board of managers, reported that the following persons, viz: C. O. C. Billberg, David Brooks, Henry Crew, Richard W. Davids, N. H. Edgerton, Richard W. Gilpin, C. Hanford Henderson, Carl Hering, Herman S. Hering, Edwin J. Houston, L. F. Rondinella, Pedro G. Salom, E. Alexander Scott, William H. Wahl, S. Lloyd Wiegand and Paul A. M. Winand, having applied to the board of managers to be constituted the electrical section of the Franklin Institute, and the said application having been approved, they are, in accordance with section 3, of article XI of the by-laws, hereby reported to the institute as the founders of the section.

He reported, further, that the following resolution was adopted by the board at its stated meeting, held December 10, 1890, viz:

"*Resolved*, that the board of managers hereby nominates to the institute for election as an honorary member, Dr. Francis Fowler, of Washington, D. C."

Dr. Fowler, thereupon, was elected as an honorary member.

The following nominations were then made for officers, managers, and members of the committee on science and the arts, viz:

For *president*	(to serve	one	year), . .	JOSEPH M. WILSON.
" *vice-president*	("	three	"), . .	WM. P. TATHAM.
" *secretary*	("	one	"), . .	WM. H. WAHL.
" *treasurer*,	("	"	"), . .	SAMUEL SARTAIN.
" *auditor*,	("	three	"), . .	WM. O. GRIGGS.

For *managers* (to serve three years):

HUGO BILGRAM,	HENRY BOWER,
CYRUS CHAMBERS, JR.,	CHAS. H. CRAMP,
G. M. ELDRIDGE,	H. W. JAYNE,
H. R. HEYL,	WM. E. LOCKWOOD,
C. H. HUTCHINSON,	FRED. B. MILES,
S. R. MARSHALL,	HENRY PEMBERTON, JR.,
C. E. RONALDSON,	H. W. SPANGLER.
WM. SELLERS,	

For members of the *committee on science and the arts* (to serve three years):

JOHN E. CODMAN,	WM. D. MARKS,
THOS. P. CONARD,	PHILIP PISTOR,
C. B. DUDLEY,	HENRY PEMBERTON, JR.,
WM. C. HEAD,	THOMAS SHAW,
C. J. HEXAMER,	L. H. SPELLIER,
H. R. HEYL,	S. P. SADTLER,
FRED. E. IVES,	T. C. SEARCH,
W. M. MCALLISTER,	WM. E. LOCKWOOD.

Mr. E. H. Johnson, of New York, read a paper describing the system of the Interior Conduit and Insulation Company, and showed the utility of the system by an experimental demonstration. (Referred for publication.)

Mr. John A. Wilson gave an account of the Philadelphia and Reading Railroad Company's terminal and elevated railway plans, describing, with the aid of colored sketches, lantern slides and detail drawings, the routes to be followed by the proposed line, and the character of the various portions of the elevated structure, buildings, crossings, etc. The subject evoked a lengthy discussion, which was participated in by Messrs. Le Van, Wiegand, Brown (Ed.), Watson, Garrison, Haupt and others. (The subject is referred for publication.)

Mr. W. N. Jennings exhibited the concluding portion of a series of photographic views, illustrating his trip to the Pacific Coast.

The secretary read the following communication, received from Mr. J. E. Watkins, of the U. S. National Museum, viz:

CIRCULAR NO. I.

Central committee for the celebration of the beginning of the second century of the American patent system by inventors and manufacturers of patented inventions.

Executive Committee.—Hon. John Lynch, Chairman; J. Elfreth Watkins, Secretary; J. W. Babson, M. C. Stone, G. C. Maynard.

—

OFFICE, BOARD OF TRADE, 1419 F STREET, N. W.
WASHINGTON, D. C., December 12, 1890.

To the inventors of America and the manufacturers of nventions:

The completion of the first century of the American patent system marks so important an epoch in the history of the nation that it is eminently proper that the beginning of the second shall not pass unnoticed.

The centennial anniversaries of other important national events have been celebrated in a manner worthy of a people proud of their country and its growth. Surely, the system that has aided the agriculturalist in the field, the mechanic in the shop, and the toiler in the mine; that has stimulated invention, and protected the rights of the inventor and his most important ally, the manufacturer, has played no small part in a history so full of the triumphs of human achievements.

Believing that the American inventor and manufacturer of inventions will regard it a privilege, as well as a duty, to co-operate in making due recognition of these facts, it is proposed to hold a celebration at the national capital, in April next, which shall in a fitting manner commemorate the important event and place on record that nation's appreciation of the labors of those whose ingenuity, patience and tireless effort have exercised such a potent influence in accelerating the prosperous growth of the nation, and in aiding the progress of our civilization.

The necessity for a national association of inventors and manufacturers of inventions organized for mutual benefit has frequently been discussed in the technical and other journals. No time could be more opportune for the formation of such an association than when men from every part of the country meet to celebrate so important an anniversary.

Surely, the occasion is most inspiring.

All inventors and manufacturers and others interested are requested to co-operate with this committee in the purpose above set forth. Correspondence appertaining thereto should be addressed to

J. ELFRETH WATKINS,
Secretary U. S. National Museum.

Mr. S. L. Wiegand spoke in favorable terms of the system of rolling steel tubes from the solid, devised by the Mannesmann brothers, and described at the previous meeting, and moved that the invention be referred to the committee on science and the arts, with the view of recognizing the merits of the same by some suitable award. Carried.

Adjourned.

WM. H. WAHL, *Secretary.*

THE OLSEN TESTING MACHINE.

Fig. 2.

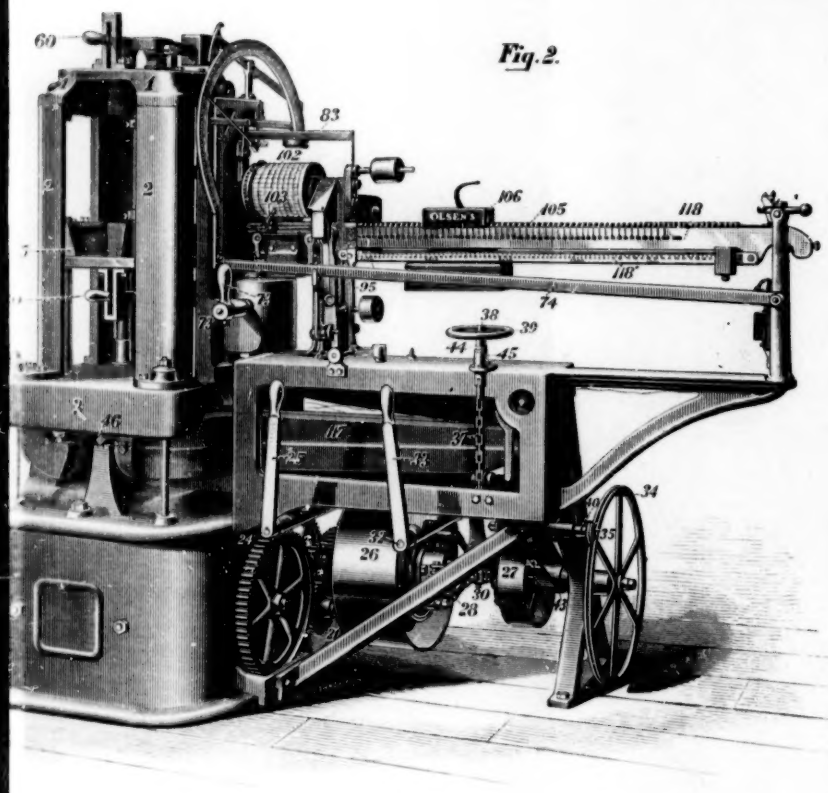
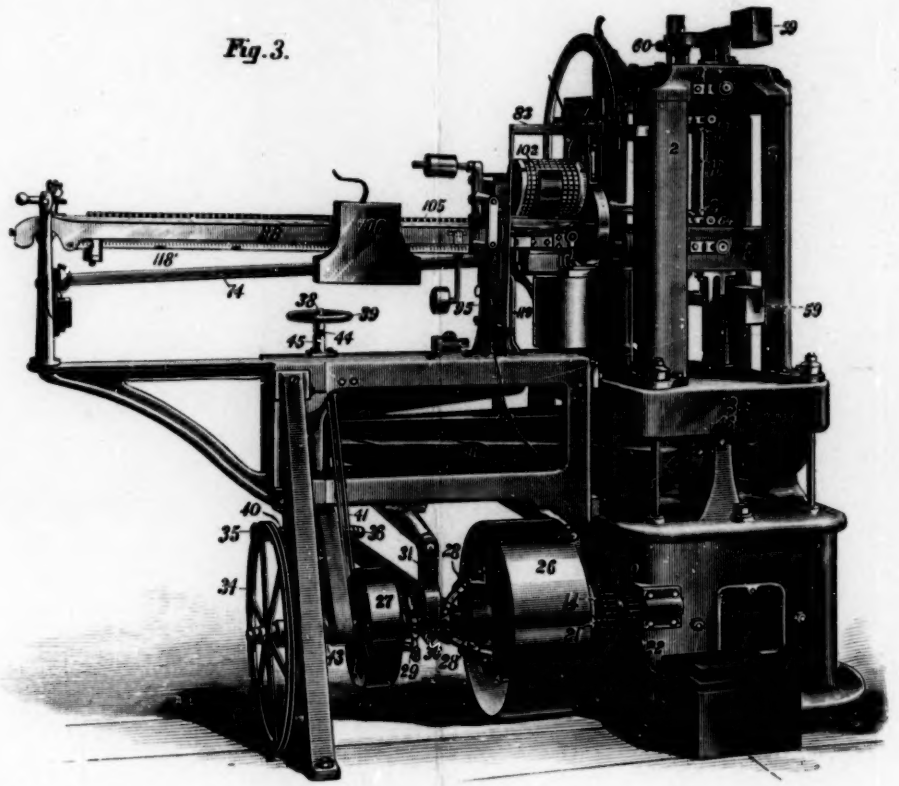


Fig. 3.



EXPLANATION OF REFERENCE NUMBERS.

ked lever controlling sleeve 30.
ft.
d lever operating 30.
oved wheel on driving shaft.
il wheel turning 34.
r to 35.
n.
isting screw.
ng bearing.
l wheel.
less band.
ey turning with pulley 27.
cal spring.
ort for 44.

46, Fulcrum of lever 117.
48, Specimen under test.
49, Gripping jaws.
50, Projecting flanges on jaws 49.
51, Block slide.
52, Grooves in 51.
53, Slotted slide supporting 49.
54, Opening in 53.
55, Eye in 53.
56, Bolt connecting 53 and 57.
57, Lever to open and shut jaws.
58, Fulcrum of 57.
59, Counterweight.
60, Handle of lever 57.
61, Plungers for slides 51.
62, Screws for 61.
63, Screw bolt.
64, Collars or clamps for caliper bearing.
65, Set-screws in 64.
66, Detachable cap on clamps 54.
67, Tenons fitting 66.
68, Guiding plates.
69, Screws to 68.
70, Nuts to screws 69.
71, Washers to 69.
72, Guiding-block.
73, Cam.
73′, Handle of cam.
73″, Grooves in clamps 64.
74, Lever moving 87.
75, Sliding-blocks.
76, Supporting-guide to 75.
77, Set-screws for 75.
78, Polygonal prism i. 75.
79, Shaft for 78.
81, Pins for holding 79.
82, 83, Calipers.
85, Arm of caliper.
86, Clamps.
87, 88, Upright bars or rods.
95, Cord operating recording cylinder.
96, Pulley.
97, Lever.
98, Fulcrum to 97.
99, Pulley or sheave.

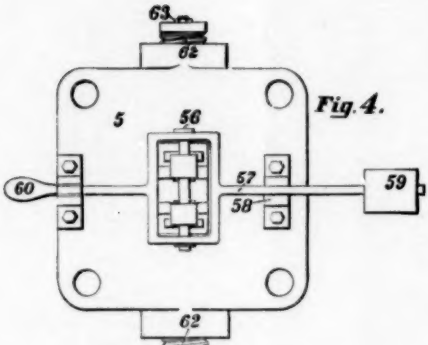


Fig. 4.

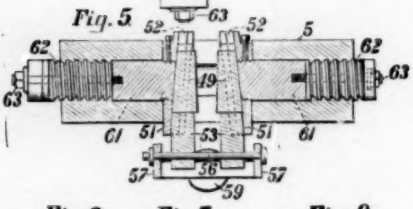


Fig. 5.

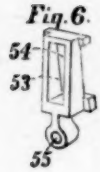


Fig. 6.

Fig. 7.

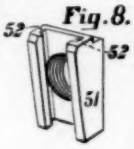


Fig. 8.

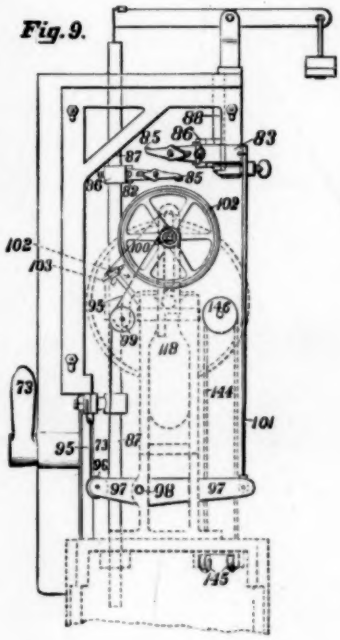


Fig. 9.

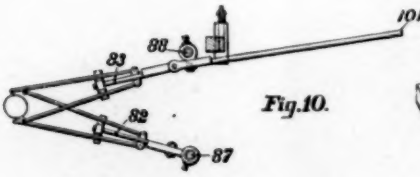


Fig. 10.

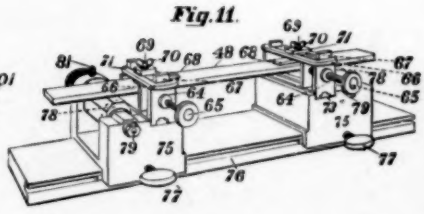
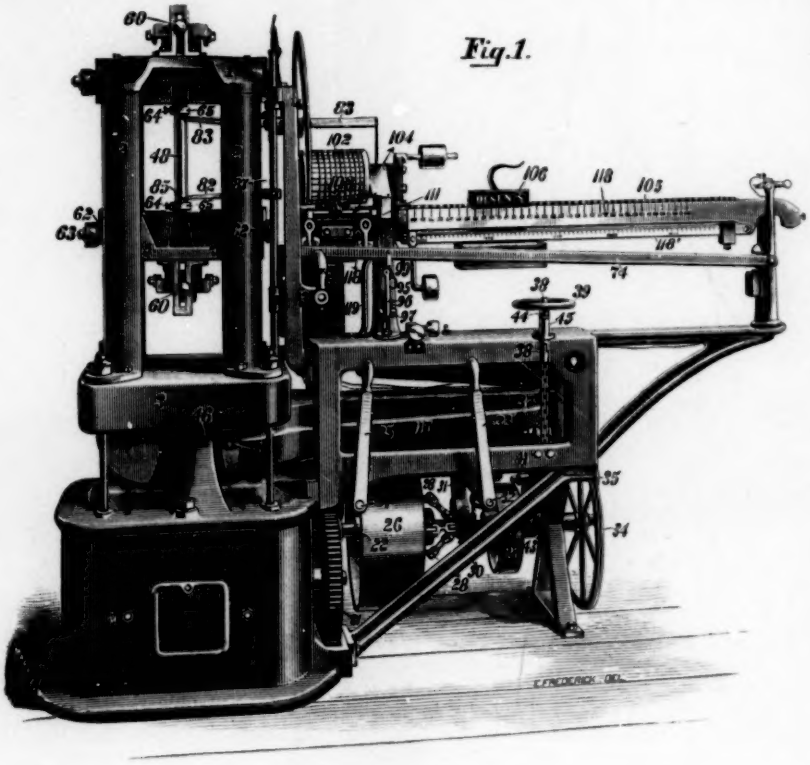


Fig. 11.

100, Drum or winding-barrel of 102.
101, Link.
102, Recording cylinder.
103, Pencil.
104, Screw.
105, Screws shifting 106.
106, Poise or weight.
111, Balancing pivot of beam.
117, Force multiplying lever.
118, Weighing beam.
118′, Slide to small poise on 118.
119, Link.
144, Endless band for moving poise.
145, Guiding pulleys.
146, Grooved wheel.

Jour.

Fig. 1.



1, Entablature.
2, Columns.
3, Platform supporting columns.
4, Pivots.
5, Lower moving head.
14, Spur wheel.
16, Idle wheel.
19, Pinion.
21, Spur wheel on sleeve 22.
22, Sleeve on driving shaft.
24, Rock shaft operating lever shifting 22.
25, Hand lever operating 24.
26, 27, Pulleys rotating driving shaft.
28, 29, Friction clutches engaging 26 with driving shaft.
30, Sleeve operating clutches.